Transportation, innovation and growth

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March 17, 2026

Abstract

This paper develops a tractable multi-industry model of endogenous growth with innovation in both manufacturing and transportation. When the per-unit cost of shipping goods is exogenous, the sales amplification of manufacturing productivity improvements is weak and growth eventually ceases. When firms allocate R&D resources to innovation in transportation, endogenous growth occurs because cost-reducing innovation in production and in transportation reinforce each other in a virtuous circle. Policies or frictions that raise per-unit transportation costs — such as cumulative regulation, infrastructure bottlenecks, or fuel price pressures — dampen the translation of productivity gains in the factory into aggregate growth. Conversely, lower per-unit transportation costs generate both level and growth effects by strengthening the ability of firms to turn process innovations into a larger volume of sales. More broadly, the results suggest that understanding growth requires treating production and distribution as an integrated system.

Keywords: endogenous growth; innovation; transportation; per-unit transportation cost; modes of transportation; regulation

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1 Introduction

A central goal of modern growth theory is to explain how economies generate sustained increases in productivity and income through endogenous technological progress. Since the seminal work of Romer (1990) and Aghion and Howitt (1992), the literature has emphasized that long-run growth arises from firms' purposeful investment in research and development (R&D). The size of markets, the degree of competition, and the ability to appropriate innovation rents jointly determine the equilibrium rate of growth. Although this literature has modeled innovation in rich detail, it has not paid much attention to the mechanisms that connect production to final demand, namely, transportation and distribution.

In the canonical Schumpeterian model, the goods that firms make reach the customers at zero transportation and distribution cost. Consequently, cost-reducing innovation in production translates directly into more sales and more profit. In reality, however, making things and shipping things are distinct and sequential activities, both necessary to serve the market. Moreover, shipping things — which henceforth we call simply transportation for brevity — consists of transportation, distribution, logistics and more, all activities that absorb real resources and are themselves subject to technological constraints. By shaping effective market size and competition, the transportation cost fundamentally conditions how innovation translates into economic growth.

In this perspective, a notable empirical pattern that standard growth models struggle to rationalize is the outsized macroeconomic impact of transportation frictions relative to distortions in other sectors. Historical episodes of transportation deregulation — such as in rail, trucking, and air freight — are associated with disproportionate gains in productivity and market expansion, while increases in per-unit transportation costs from regulation or infrastructure bottlenecks generate unusually large welfare and growth losses. This asymmetry reflects the fact that transportation is not merely another industry but a necessary input into all production and exchange, a feature that our model captures by endogenizing the interaction between manufacturing innovation and transportation costs

The literature has identified three closely linked channels through which transportation affects innovation-led growth. First, **market size and competition**: lower transport cost expands the reach of firms and raise potential returns from R&D by enlarging the customer base, but they also intensify competition as more producers can serve overlapping markets, compressing markups and potentially reducing R&D incentives (Desmet and Rossi-Hansberg 2014). The net effect on growth, therefore, depends on the relative strength of market-size and competition effects. Second, **diffusion and spillovers**: improved transportation infrastructure accelerates the spatial spread of technologies, knowledge, and intermediate goods. Comin, Dmitriev and Rossi-Hansberg (2012) show empirically that geographic distance and connectivity strongly affect the speed at which innovations propagate across space. Faster diffusion broadens the base of adopters and increases aggregate productivity, though it may also intensify creative destruction locally. Third, **iceberg vs per-unit cost**: transportation is a distinct economic activity with its own technology and the form of its cost, iceberg vs per-unit, determines how innovation in production translates into more output delivered to the customer. With an iceberg cost of shipping things, productivity gains in the factory (making things) pass through fully to consumers. With a per-unit cost, part of the productivity gain in the factory is dissipated by delivery expenses, weakening the link between process innovation and sales. This distinction, formalized in the famous Alchian-Allen hypothesis that a per-unit tariff or shipping cost reduces the relative price of expensive goods (Alchian and Allen 1964), has profound implications for the economic mechanism that links transportation to economic growth.[1]

In a recent paper, Kane and Peretto (2020) take the Alchian-Allen hypothesis as their starting point and develop a framework in which the structure of the transportation and distribution technology governs the composition of innovation-led growth. With iceberg shipping costs, the economy sustains steady-state growth through both cost-reducing innovation — more apples — and quality-improving innovation — better apples. With per-unit distribution costs, in contrast, the economy sustains steady-state growth only through quality-improving innovation. The reason is that iceberg costs are in units of the good that the firm makes, which implies that higher productivity in making things is also higher productivity in shipping things. Per-unit shipping costs do not have this property: they do not fall with innovation in the factory. As a consequence, they weaken or even break altogether the link between process innovation and the growth of the firm's sales. The framework thus bridges the theory of innovation-driven growth with the economics of logistics.

In this paper, we build on that insight but abstract from quality improvements to focus exclusively on the interaction between cost-reducing and variety-expanding innovations. By eliminating the quality margin, we isolate the mechanism that links process innovation, distribution technology, and aggregate growth. This focus corresponds to many manufacturing

[1]It is also empirically valid (Hummels and Skiba 2004) and relevant to welfare calculations. Sørensen (2014) and Irarrazabal et al. (2015), for example, show that increases in per-unit frictions lead to higher welfare losses compared to increases in iceberg frictions in trade models.

and intermediate-goods industries where innovation primarily takes the form of productivity improvement and product diversification.

Our model economy consists of an integer number of industries, each one populated by an endogenous mass of firms that produce differentiated intermediate goods. The typical firm in the typical industry makes a differentiated good and then ships it to the customers. To ship a unit of its good the firm incurs a transportation that consists of an iceberg component and a per-unit component. Absent innovation in transportation, the per-unit component acts as an incompressible wedge between factory and market prices.[2] Even if the firm continually innovates to reduce the production cost, the per unit delivery cost limits the firm's expansion. As a result, the firm faces economic diminishing returns to process innovation: even though the firm's cost-reducing innovation technology has the properties of the standard Schumpeterian cost-reduction endogenous growth model (e.g., Peretto 1998, 1999, 2025), the rate of return to process innovation falls over time as the factory price falls to zero while the market price converges to a positive lower bound determined by the per-unit transportation cost. This lower bound prevents the firm from moving down the demand curve all the way to infinite sales as in the standard cost-reduction model. The economy then converges to a steady state in which firms do not do R&D. To obtain endogenous growth in this world, therefore, firms must invest in transportation-specific innovation. When firms (or a specialized transportation sector) invest in R&D that reduces the per-unit shipping cost, productivity improvements in transportation amplify those in manufacturing. The two forms of innovation become mutually reinforcing, yielding a balanced growth path in which both activities expand at the same constant rate. In this equilibrium, the rate of growth depends on parameters governing the cost of R&D, the elasticity of substitution among intermediate goods, and the technological linkages between production and transport.

The main advantage of elminating the quality margin is that we obtain a very tractable multi-industry model that we can solve analytically. We can thus study the growth and welfare effects of changes in the industry-specific structural parameters of the model accounting for the full transitional dynamics. We can also extend the model to a richer characterization of production and transportation that allows for fuel as an additional input and for multiple modes of transportation (e.g., air, water, truck, rail). In this extension, firms chose optimally the mix of labor and fuel used to make and ship things and the mix of modes

[2] This wedge is empirically important. Burstein et al. (2003), for example, show that distribution costs account for over 40% of the retail price of a typical consumer good in the United States and over 60% in Argentina.

that they use to ship things. Finally, we can study the secular transition of the economy from a pre-industriual state to the industrial state and the subsequent convergence to the balanced-growth path. Overall, the analysis produces four principal results.

Necessity of transportation innovation. When the transportation technology is fixed, the per-unit transportation cost imposes a lower bound on the firm's market price that ultimately nullifies the effect of process innovation and thus halts endogenous growth.

Complementarity between manufacturing and transportation R&D. Innovation in transportation removes the lower bound on the market price and thus allows sales to grow to infinity, restoring enodgenous growth. Moreover, reductions in per-unit delivery costs raise the profitability of manufacturing R&D, and vice versa, producing co-evolution of the two technologies in a virtuous circle.

Structural interpretation of growth accelerations and slowdowns. The model provides a framework to interpret observed accelerations and slowdowns in productivity as the consequence of falling or rising transportation costs — whether due to regulatory accumulation, infrastructure bottlenecks, or energy constraints — that strenghten or weaken the translation of micro-level innovations into aggregate outcomes.

Heterogeneity across industries. The mix of transportation modes and the interaction of innovation in making things and in shipping things are industry-specific. Despite this heterogeneity, however, the model aggregates nicely, producing transparent results for aggregate growth and welfare.

This paper connects two strands of research that have largely evolved separately. The first is the endogenous-growth literature, which has extensively analyzed the incentives for innovation, the allocation of R&D resources to different types of innovation, and the welfare implications of market structure (Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992, 1998, 2009, Peretto 1998, 1999, 2025, Acemoglu 2009). These models typically assume frictionless delivery and abstract from transportation, distribution and logistics. The second is the spatial and transportation economics literature, which studies how trade costs, infrastructure, and spatial frictions affect economic geography and productivity (Krugman 1991, Fujita and Thisse 2002, Desmet and Rossi-Hansberg 2014). Recent work at the intersection of these literatures — such as Kane and Peretto (2020) — has begun to integrate distribution into growth models explicitly, showing that the cost structure of transportation can change the nature of the innovation engine itself.

Empirically, numerous studies show that transportation infrastructure and logistics performance are correlated with GDP and total factor productivity (Fogel 1964, Aschauer 1989, Fernald 1999, Calderón and Servén 2010, Allen and Arkolakis 2014, Donaldson and Hornbeck 2016, Acheampong et al. 2021). At the same time, evidence on diffusion (Comin et al. 2012) suggests that connectivity shapes not only levels but also the speed of technological progress. These findings reinforce the need for theoretical frameworks that incorporate transportation as an important component of the growth process.

To summarize, the contribution of this paper is threefold. First, it develops a tractable multi-industry endogenous-growth model in which transportation is a distinct economic activity that absorbs resources and requires technological progress to sustain growth. This formulation endogenizes the link between logistics and innovation and clarifies how per-unit transportation costs shape economic growth. Second, it derives analytical conditions for endogenous growth in the joint presence of manufacturing and transportation innovation, providing explicit comparative statics and dynamics on R&D, growth and welfare. Third, it offers a structural interpretation of growth accelerations and slowdowns as manifestations of, respectively, improving or worsening productivity in transportation, thereby bridging the gap between macro-productivity trends and micro-level logistical frictions.

The results have direct policy implications. Because transportation costs determine how efficiently innovations in production reach markets, investments in infrastructure and logistics R&D can have both level and growth effects. Policies that ignore this complementarity risk overestimating the returns to manufacturing innovation while underinvesting in the distribution technologies that enable those innovations to scale into larger sales. Conversely, reforms that reduce regulatory or energy burdens on transportation can amplify the economy's capacity for sustained, innovation-driven growth. More broadly, the results suggest that understanding growth requires treating production and distribution as an integrated system. Future research could extend the framework to include quality improvements, endogenous infrastructure investment, or international linkages, thereby providing a richer theory of how innovation and transportation jointly determine long-run prosperity.

The remainder of the paper is organized as follows. Section 2 presents the model environment. Section 3 describes production and R&D decisions, their relation to the structure of transportation costs, and defines the economy's general equilibrium. Section 4 analyzes the multi-industry general equilibrium dynamics under exogenous transport technology. Section 5 introduces innovation in transportation. Section 6 studies the model's comparative statics and dynamics properties, developing the paper's main growth, welfare and policy implications. Section 7 develops a richer representation of the economy, introducing fuel as an additional input and allowing for two modes of transportation. Section 8 studies the model's implications for secular growth. Section 9 concludes.

2 Model

In this section we set up the model and discuss its conceptual and analytical foundations.

2.1 Model structure

The production side of the economy consists of a representative competitive firm that combines intermediate goods to produce a homogeneous final good. To obtain a multi-industry structure, we model this production process as consisting of two vertically stacked layers.

At the bottom there is a Cobb-Douglas aggregator representing production of the final good as the assembly of differentiated components. Next up there is the second layer, in which each one of the differentiated components used in layer one is produced by a competitive firm operating a CES aggregator of specialized intermediate goods. Therefore, in this model an industry consists of an endogenous set of specialized firms that supply the intermediate inputs used in the production of one of the differentiated components of the homogeneous consumption good. In both layer one and layer two of the consumption good sector, production is the assembly of differentiated inputs according to well-understood aggregators.

Each intermediate firm belongs to a specific industry endowed with its own industry-specific parameters. To keep things simple, we assume that the labor market is fully integrated with mobility across industries so that all firms in all industries face the same wage. Similarly, the financial market is fully integrated so that all firms in all industries face the same interest rate.

2.2 Household

The representative household has $L(t) = \Lambda e^{\lambda t}$ members, where $\Lambda > 0$ and $\lambda \in (-\infty, +\infty)$. In this setup, the size of the household, L, is population, where Λ is a scale factor and λ is the growth rate of the population. We do not restrict λ to be positive. Finally, to keep things simple, we abstract for labor-leisure choice so that the household's labor supply is L.

The household maximizes lifetime utility

$$U(t) = \int_t^{\infty} e^{-\rho(s-t)} L(s)\, u(s) ds, \quad \rho > \max\{0, \lambda\}, \tag{1}$$

where ρ is the discount rate. Instantaneous utility is

$$u = \log\left(\frac{Y}{p_G L}\right), \tag{2}$$

where Y is the household's expenditure on the consumption good whose price is p_G. The household's budget constraint is

$$\dot{A} = rA + \underbrace{\Pi_G + \sum_{j=1}^{J} \Pi_{G_j}}_{\text{both 0 in equilibrium}} + wL - Y, \tag{3}$$

where A is assets holding (equity shares in intermediate firms), r is the rate of return on assets, w is the wage, and Π_G and Π_{G_j} are the profits distributed to the household as dividends by the consumption good sector and its suppliers of differentiated components $j = 1, ..., J$. These terms are zero in equilibrium.

The household chooses the time path of expenditure Y to maximize (1) subject to (2) and (3). The solution to this problem yields the Euler equation governing saving behavior,

$$r = r_A \equiv \rho + \frac{\dot{Y}}{Y} - \lambda. \tag{4}$$

We interpret r_A as the reservation rate of return on financial assets at which the household is willing to trade current for future consumption.

2.3 Consumption good producer

In layer one, a representative competitive firm produces the homogeneous consumption good with the Cobb-Douglas technology

$$G = J \prod_{j=1}^{J} G_j^{\varphi_j}, \quad \text{with} \quad \sum_{j=1}^{J} \varphi_j = 1. \tag{5}$$

This is a well understood structure: the competitive layer-one firm spends on each component j a fraction $Y_j/Y = \varphi_j$ of its revenues, $Y = p_G G$. Moreover, in competitive equilibrium this producer makes zero profits, i.e., $\Pi_G = 0$.

The typical competitive layer-two firm operates the CES technology

$$G_j = N_j^{\chi_j - \frac{1}{\epsilon_j - 1}} \left(\int_0^{N_j} X_{ij}^{\frac{\epsilon_j - 1}{\epsilon_j}} di \right)^{\frac{\epsilon_j}{\epsilon_j - 1}}, \quad \chi_j > 0, \ \epsilon_j > 1, \quad j = 1, ..., J, \tag{6}$$

where χ_j is the degree of increasing returns to variety, ϵ_j is the elasticity of product substitution, X_{ij} is the quantity of the non-durable intermediate good i supplied to industry j, and N_j is the mass of suppliers active in industry j. This structure is also well understood and delivers the demand system for intermediate goods in industry j,

$$X_{ij} = Y_j \frac{p_{X_{ij}}^{-\epsilon_j}}{\int_0^{N_j} p_{X_{kj}}^{1-\epsilon_j} dk}, \quad i \in [0, N_j], \quad j = 1, ..., J. \tag{7}$$

Each layer-two firm makes zero profits in competitive equilibrium. Therefore, $\sum_{j=1}^{J} \Pi_{G_j} = 0$.

2.4 Production, transportation, innovation

In this section we characterize intermediate firms and the technologies that they use to make things and to ship things. Understanding the tight interdependence between making things and shipping things is the main objective of the paper. We start with a simple description of the relationship that does not model explicitly the existence of different modes of transportation in order to focus on the model's general equilibrium structure and the dynamics of innovation and market structure that it produces. In section 7, we develop an extension that allows for different modes of transportation and characterize the industry-specific combination of modes that emerges in market equilibrium.

2.4.1 Technologies

Firm i in industry j operates four technologies: the manufacturing technology to make the good, the transportation technology to ship the good, the manufacturing innovation technology to figure out ways to improve the productivity of labor in making the good, and the transportation innovation technology to figure out ways to improve the productivity of labor in shipping the good.

2.4.2 Manufacturing

The firm produces its differentiated intermediate good with the technology

$$X_{ij} = Z_{ij}^{\theta_j}\left(L_{X_{ij}} - \phi_j\right) \quad 0 < \theta_j < 1,\ \phi_j > 0, \tag{8}$$

where X_{ij} is output, $Z_{ij}^{\theta_j}$ is labor productivity as a function of the stock of firm-specific manufacturing knowledge, $L_{X_{ij}}$ is total manufacturing labor and ϕ_j is overhead labor.

2.4.3 Transportation

The firm must ship its production to the customers. We restrict attention to a transportation technology with labor as the single input and assume in-house distribution; we can incorporate in the model a separate competitive market for distribution services without changing the results. We posit the following transportation technology

$$L_{D_{ij}} = \left[\underbrace{\tau_j Z_{ji}^{-\theta_j}}_{\text{iceberg}} + \underbrace{(1+\tau_j)\,\eta_j S_j^{-\sigma_j}}_{\text{per-unit}}\right] X_{ij}, \quad \eta_j \geq 0,\ \sigma_j \geq 0 \tag{9}$$

where $L_{D_{ij}}$ is the amount of labor required to ship X_{ij} units of good ij to the customer, η_j and σ_j are parameters and S_j is the stock of firm-specific transportation knowledge. Accordingly, the cost of shipping one unit to the customer consists of an iceberg component and a per-unit component. The iceberg component consists of the cost of producing the additional amount of the good needed to make up for the anticipated melting during shipping. The per-unit component consists of the cost of physically moving the good regardless of whether it melts on the way. Overall, the cost of shipping a good is the sum of the cost of producing the anticipated melted amount and the per-unit cost of delivering the (unmelted) good and the anticipated melted amount.

We allow for both the iceberg and the per-unit components to study their different implications and to connect our paper to the existing literature in which iceberg costs play a major role. To understand better this structure, it is useful to contrast the cases $\eta_j = 0$, $\tau_j > 0$ and $\eta_j > 0$, $\tau_j = 0$. When the distribution technology consists only of the iceberg cost, an increase in manufacturing productivity reduces the labor needed to produce the extra quantity that melts in transit, thereby reducing the cost of shipping. When the distribution technology consists only of the per-unit cost, in contrast, a reduction in production cost does not yield a reduction in distribution cost. To highlight the role of transportation even more, we study the cases $\sigma_j = 0$ and $\sigma_j > 0$ separately. With $\sigma_j = 0$, the per-unit cost is completely exogenous and chokes off endogenous growth even though the manufacturing innovation technology has the technical properties that would allow it. With $\sigma_j > 0$, the per-unit cost is endogenous and endogenous growth occurs because innovation in transportation *complements* innovation in manufacturing. To the best of our knowledge, this form of innovation-based dynamic complementarity is new to the literature.

The key to this formulation is that making things and shipping things are separate activities, each one with its own technology, that are complementary. The iceberg component allows for a "spillover" from innovation in manufacturing to shipping, but such spillover is not the whole story. The per-unit component entails an incompressible cost of shipping that has qualitatively important effects for innovation. "Incompressible" here means that the firm's effort in improving labor productivity in making things cannot drive to zero the price of the good for the customer — as in traditional endogenous technological change models —because the price contains the per unit cost of shipping things, which does not fall proportionally to the cost of making things. In this sense, our model articulates the idea that endogenous growth is a property of the integrated "manufacturing + transportation" system and cannot be understood by looking at one component in isolation from the other.

2.4.4 Manufacturing innovation

The firm accumulates manufacturing knowledge according to

$$\dot{Z}_{ij} = \alpha_j Z_j L_{Z_{ij}}, \quad \alpha_j > 0, \tag{10}$$

where $\dot{Z}_{ij}$ is the flow of new firm-specific manufacturing knowledge generated by $L_{Z_{ij}}$ units of R&D labor with productivity $\alpha_j Z_j$, where α_j is an exogenous parameter and Z_j is the stock of knowledge available to and useful to the firm in R&D operations. To obtain a tractable structure, we assume that this spillover consists of the average knowledge developed by all firms *in the industry*. In particular, we assume

$$Z_j = \int_0^{N_j} \frac{Z_{sj}}{N_j} ds,$$

where N_j is the mass of firms in industry j.

2.4.5 Transportation innovation

To identify as cleanly as possible the interdependence of manufacturing and shipping as a market equilibrium phenomenon, we want the transportation innovation technology to be as similar as possible to the manufacturing innovation technology. Therefore, the firm accumulates transportation-specific knowledge according to

$$\dot{S}_{ij} = \varsigma_j S_j L_{S_{ij}}, \quad \varsigma_j > 0, \tag{11}$$

where $\dot{S}_{ij}$ is the flow of new firm-specific transportation-specific knowledge generated by $L_{S_{ij}}$ units of R&D labor with productivity $\varsigma_j S_j$, where ς_j is an exogenous parameter and S_j is the stock of general-purpose transportation knowledge available to and useful to the firm in this activity. As for manufacturing, we assume that this spillover consists of the average knowledge developed by all firms *in the industry* and assume

$$S_j = \int_0^{N_j} \frac{S_{kj}}{N_j} dk,$$

where N_j is the mass of firms in industry j.

3 Firm behavior, entry and general equilibrium

In this section we characterize the decisions of intermediate firms and entrants and construct the economy's general equilibrium.

3.1 Firm decisions

At time t, firm i maximizes the present discounted value of dividends,

$$V_{ij}(t) = \int_t^\infty e^{-\int_t^s [r(v)+\delta_j]v} \Pi_{ij}(s) ds, \quad \delta_j > 0, \tag{12}$$

where the firm's dividend flow is

$$\Pi_{ij} = \left[p_{X_{ij}} - w(1+\tau_j)\left(Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j} \right) \right] X_{ij} - w\phi_j - wL_{Z_{ij}} - wL_{S_{ij}} \tag{13}$$

and δ_j is a death shock.[3] With our assumption of a continuum of intermediate goods, each intermediate firm is atomistic. Therefore, the firm maximizes (12) subject to the demand curve (7), the innovation technology (10), the non-negativity constraints on R&D labor, $L_{Z_{ij}} \geq 0$ and $L_{S_{ij}} \geq 0$, taking as given the initial stock of manufacturing knowledge, $Z_{ij}(t) > 0$, and the initial stock of transportation knowledge, $S_{ij}(t) > 0$. The firm also takes as given the paths of spillovers, Z_j and S_j, and of the mass of firms, N_j. The solution to the firm's problem yields the maximized value of the firm.

To characterize the firm's behavior, we write the Current Value Hamiltonian

$$\begin{aligned} CVH_i &= \left[p_{X_{ij}} - wZ_i^{-\theta} - w\tau_j Z_{ij}^{-\theta_j} - w\left(1+\tau_j\right)\eta_j S_{ij}^{-\sigma_j}\right] X_{ij} - w\phi_j \\ &\quad - wL_{Z_{ij}} - wL_{S_{ij}} + \upsilon_{Z_{ij}}\alpha_j Z_j L_{Z_{ij}} + \upsilon_{S_{ij}}\varsigma_j S_j L_{S_{ij}}, \end{aligned}$$

where the costate variable, $\upsilon_{Z_{ij}}$, is the shadow value of the marginal unit of manufacturing knowledge and the costate variable, $\upsilon_{S_{ij}}$, is the shadow value of the marginal unit of transportation-specific knowledge. The firm's manufacturing knowledge stock, Z_{ij}, is the state variable that evolves according to equation (10) and the firm's transportation knowledge, S_{ij}, is the state variable that evolves according to equation (11). The product's price, $p_{X_{ij}}$, and R&D labor, $L_{Z_{ij}}$ and $L_{S_{ij}}$, are the control variables. Given the properties of our primitives, the first-order conditions of this problem are sufficient for maximization and deliver the following relations.

The firm's value-maximizing pricing decision is

$$p_{X_{ij}} = \frac{\epsilon_j}{\epsilon_j - 1} w\left(1+\tau_j\right)\left(Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}\right). \tag{14}$$

Inverting the production technology (8) and using the price strategy (14) yields again the firm's conditional demand (15) for manufacturing and transportation labor

$$L_{X_{ij}} + L_{D_{ij}} = \frac{\epsilon_j - 1}{\epsilon_j}\frac{p_{X_{ij}} X_{ij}}{w} + \phi_j. \tag{15}$$

[3] The exit shock guarantees that the model has symmetric dynamics in a neighborhood of the steady state when population growth is zero. It also provides additional empirical discipline in quantitative exercises; see, e.g., Ferraro, Ghazi and Peretto (2020) and (2023).

We can break it down in:

$$L_{X_{ij}} = \frac{\epsilon_j - 1}{\epsilon_j} \frac{Z_{ij}^{-\theta_j}}{(1+\tau_j)\left(Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}\right)} \frac{p_{X_{ij}} X_{ij}}{w} + \phi_j;$$

$$L_{D_{ij}} = \frac{\epsilon_j - 1}{\epsilon_j} \frac{\tau_j Z_{ji}^{-\theta_j} + (1+\tau_j)\,\eta_j S_{ij}^{-\sigma_j}}{(1+\tau_j)\left(Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}\right)} \frac{p_{X_{ij}} X_{ij}}{w}.$$

The firm also employs labor in two R&D activities, $L_{Z_{ij}}$ and $L_{S_{ij}}$, to which we now turn.

Because the Hamiltonian is linear in $L_{Z_{ij}}$ and in $L_{S_{ij}}$, the investment decisions have a bang-bang structure. We provide the details in the appendix. Here we simply report that the investment decisions are summarized by two equations that describe, respectively, the rate of return to in-hous innovation in production and the rate of return to in-house innovation in transportation. Next, we note that Peretto (2025) explains that under the restriction $1 > \theta_j (\epsilon_j - 1)$ the industry equilibrium is symmetric because (1) firms make identical price and R&D decisions, (2) we start them with identical values of initial firm-specific manufacturing and of transportation knowledge and, as we discuss below, (3) we assume that new firms enter at the average level of firm-specific manufacturing and transportation knowledge. Consequently, we can drop the subscript i from our firm-specific variables with the understanding that they denote the average across firms. For example, L_{X_j} denotes the average employment of labor in manufacturing across the N_j firms, L_{Z_j} denotes the average employment of labor in manufacturing-specific R&D across the N_j firms, and so on. In particular, since $Y_j = \int_0^{N_j} p_{X_{ij}} X_{ij} di$ we write $p_{X_j} X_j = Y_j / N_j$, where X_j is the average production across the N_j firms. Therefore, equation (15) yields

$$L_{X_j} + L_{D_j} = \frac{\epsilon_j - 1}{\epsilon_j} \frac{Y_j}{w N_j} + \phi_j, \tag{16}$$

where Y_j / N_j is average sales across the N_j firms.

The rates of return to innovation are:

$$r = r_{Z_j} \equiv \alpha_j \left[\frac{Y_j}{w N_j} \frac{\theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{Z_{ij}^{-\theta_j}}{Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}} - L_{Z_j} \right] + \frac{\dot{w}}{w} - \delta_j; \tag{17}$$

$$r = r_{S_j} \equiv \varsigma_j \left[\frac{Y_j}{w N_j} \frac{\sigma_j (\epsilon_j - 1)}{\epsilon_j} \frac{\eta_j S_{ij}^{-\sigma_j}}{Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}} - L_{S_j} \right] + \frac{\dot{w}}{w} - \delta_j. \tag{18}$$

These expressions capture a core property of our theory of endogenous innovation in manufacturing and transportation built on the existence of increasing returns to scale internal to the firm: the rate of return to firm-specific innovation is a linear function of the size of the firm, given by the size of the market in which the firm operates times the market share that the firm secures. The market share channel captures another core property of our theory of endogenous innovation, which is built on the idea that firms compete in markets whose structure in the Industrial Organization sense — number of firms, average firm size, concentration and so on — is endogenous because of entry and exit.

3.2 Entry

An entrepreneur can create a new firm that starts with productivity equal to the industry average by using $\beta_j Y_j / w N_j$ units of labor.[4] The sunk entry cost, therefore, is industry-specific because it depends on the average size of the firms in the industry. In particular, we have $wL_{N_{ji}} = \beta_j Y_j / N_j$. Once in the market, the entrant firm solves a problem identical to the one outlined above for the incumbent firm. For each industry j, there exists an equilibrium with no entry when $V_j < \beta_j Y_j / N_j$ and there exists an equilibrium with entry when $V_j = \beta_j Y_j / N_j$. The associated rate of return to entry is

$$r = r_{N_j} \equiv \frac{N_j}{\beta_j Y_j}\left(\frac{Y_j}{\epsilon_j N_j} - w\phi_j - wL_{Z_j} - wL_{S_j}\right) + \frac{\dot{Y}_j}{Y_j} - \frac{\dot{N}_j}{N_j} - \delta_j. \tag{19}$$

The possibility of an equilibrium with no entry applies to each industry j independently of what happens in the other industries. Thus, this economy admits the coexistence of industries in no firm entry equilibrium with industries in free-entry equilibrium.

3.3 General equilibrium

We choose labor as the numeraire and set $w \equiv 1$. Assets market equilibrium requires:

- equalization of *all* rates of return, that is, $r = r_A = r_{Z_j} = r_{N_j}$ for all $j = 1, ..., J$;

[4] See Peretto and Connolly (2007) and Peretto (2025) for an interpretation of this assumption and alternative formulations of the sunk entry cost that leave the fundamental equilibrium properties of the theory unchanged. Also, in this class of general equilibrium models, an entrepreneur is any member of the representative household who chooses to assume the role by issuing equity in the financial market to raise the resources needed to hire labor to develop a new good and set up the firm that will bring it to the market.

- the value of the household's portfolio equals the value of securities issued by firms, $A = \sum_{j=1}^{J} N_j V_j$.

The standard no-arbitrage argument says that if any of the returns is below the reservation rate of return of savers, the associated activity is return dominated and shuts down. Consequently, the general equilibrium has two possible configurations.

1. Free entry holds in *all* industries.

2. Free entry fails in *at least one* industry.

We concentrate on configuration 1 and leave configuration 2 to future work.[5]

When the free-entry condition holds in all industries, we have

$$A = \sum_{j=1}^{J} N_j V_j = \sum_{j=1}^{J} N_j \beta_j \frac{Y_j}{N_j} = \sum_{j=1}^{J} \beta_j \varphi_j Y = \beta Y,$$

where $\beta \equiv \sum_{j=1}^{J} \beta_j \varphi_j$. This is a nice aggregation result: the economy features a constant wealth/expenditure ratio despite the heterogeneity across industries. Substituting this result in the household budget constraint (3) and using the saving rule (4) yields

$$\frac{\dot{Y}}{Y} = \rho + \frac{\dot{Y}}{Y} - \lambda + \frac{wL - Y}{\beta Y}.$$

As expenditure growth cancels out, this equation yields

$$Y = \left[1 - (\rho - \lambda)\beta\right]^{-1} \cdot wL, \tag{20}$$

where we assume $1 > (\rho - \lambda)\beta$ to guarantee a positive solution. This result says that consumption expenditure is proportional to wage income. Recalling the normalization $w \equiv 1$, expenditure per capita becomes

$$y = y^* \equiv \left[1 - (\rho - \lambda)\beta\right]^{-1}. \tag{21}$$

[5] Specifically, we plan to study structural slumps and a novel phenomenon specific to the multi-industry setting of this class of models, that is, a strong form of propagation of local events to the whole economy that we label *contagion*; see Peretto (2025) for the definition and analysis of these elements in models with no transportation.

Since expenditure per capita is constant, the saving rule (4) yields that the interest rate is $r = \rho$. Note that establishing this result does not require taking a stand on whether incumbent firms do or do not do in-house R&D.

Given this aggregation results, a self-contained system governs the dynamics of each industry when free entry holds in all industries. This property makes the model analytically tractable.

4 No innovation in transportation

In this section we study the model with no innovation in transportation. In particular, we set $\sigma_j = 0$ for all j so that every industry in the economy is at the corner solution $L_{S_j} = 0$ discussed in the previous section. The benefit of discussing this special case is that it reveals a fundamental property of the interdependence between making things and shipping things that the model captures. Specifically, the existence of the per-unit cost of shipping things kills endogenous growth in a model that seemingly possesses all the elements required to deliver it. In other words, the introduction of transportation in an otherwise conventional model of innovation-led endogenous growth alters drastically the *qualitative* properties of the growth process that the model describes.

4.1 Industry-specific dynamics

The market size of industry j is $Y_j = \varphi_j Y^*$, where $Y^* = Ly^*$. The associated employment share is $L_j/L = \varphi_j$. We define industry-specific firm size, $x_j \equiv Y_j/N_j$, and solve equation (17) for

$$z_j \equiv \frac{\dot{Z}_j}{Z_j} = \alpha_j L_{Z_j} = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j}{1+q_j} - \rho - \delta_j, \tag{22}$$

where we define the composite variable

$$q_j \equiv \eta_j Z_j^{\theta_j} S_{ij}^{-\sigma_j} = \frac{Z_j^{\theta_j}}{S_{ij}^{\sigma_j}/\eta_j} = \frac{\text{labor productivity in manufacturing}}{\text{labor productivity in shipping}}, \tag{23}$$

which in this section reduces to $q_j = \eta_j Z_j^{\theta_j}$.

Equation (22) says that firms in the industry innovate if

$$x_j > x_{Z_j}(q_j) \equiv \frac{\epsilon_j (\rho + \delta_j)}{\alpha_j \theta_j (\epsilon_j - 1)} (1 + q_j).$$

We then substitute this result in equation (19) to write the dynamical system:

$$\frac{\dot{q}_j}{q_j} = \theta_j z_j = \theta_j \left[\frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j}{1+q_j} - \rho - \delta_j \right]; \tag{24}$$

$$\frac{\dot{x}_j}{x_j} = \rho + \delta_j - \frac{1}{\beta x_j} \left[\frac{x_j}{\epsilon_j} - \phi_j - \frac{\theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j}{1+q_j} + \frac{\rho + \delta_j}{\alpha_j} \right]. \tag{25}$$

The second equation applies if agents are willing to finance entry, that is, if

$$x_j > x_{N_j}(q_j) \equiv \frac{\phi_j - \frac{\rho+\delta_j}{\alpha_j}}{\frac{1}{\epsilon_j} - \frac{\theta_j(\epsilon_j - 1)}{\epsilon_j (1+q_j)} - (\rho - \lambda)\beta}.$$

The associated rate of entry is

$$n_j \equiv \frac{\dot{N}_j}{N_j} = \frac{1}{\beta x_j} \left[\frac{x_j}{\epsilon_j} - \phi_j - \frac{\theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j}{1+q_j} + \frac{\rho + \delta_j}{\alpha_j} \right] - \lambda - \rho - \delta_j.$$

Otherwise, we have:

$$\frac{\dot{x}_j}{x_j} = \rho + \delta_j - \frac{1}{\beta x_j} \left(\frac{x_j}{\epsilon_j} - \phi_j \right);$$

$$n_j = \frac{1}{\beta x_j} \left(\frac{x_j}{\epsilon_j} - \phi_j \right) \lambda - \rho - \delta_j$$

This system (24)-(25) does not have a steady state with endogenous growth if $\eta_j > 0$ because as long as firms invest and $z_j > 0$, the variable q_j grows and drags to zero the return to manufacturing innovation. The reason is that the cost of shipping the good to the customers does not shrink to zero in lockstep with the cost of making the good. The price of the good, therefore, cannot go to zero as in standard models of endogenous growth in which the economy moves all the way to infinite sales along an isoelastic demand curve. A steady state with endogenous growth exists only for $\eta_j = 0$, i.e., there is no per-unit transportation cost. Under this assumption, equation (24) degenerates to $\dot{q}_j = 0$ for any value of z_j and we are left with a traditional model of endogenous growth.

To characterize the dynamics, we look at the locus

$$\frac{\dot{x}_j}{x_j} \geq 0: \quad x_j \leq \frac{\left(\phi_j - \frac{\rho+\delta_j}{\alpha_j} \right)(1+q_j)}{\left[\frac{1}{\epsilon_j} - \beta(\rho + \delta_j) \right](1+q_j) - \frac{\theta_j(\epsilon_j - 1)}{\epsilon_j}}$$

and construct the phase diagram in Figure 1. We have a region of path dependence and a

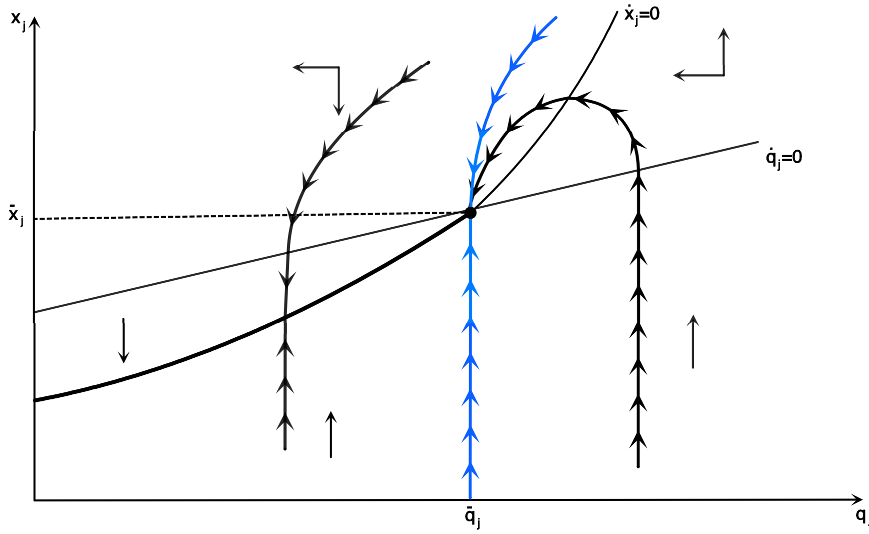

Figure 1: Equilibrium dynamics with no innovation in transportation.

region of trajectories that for given initial condition converge to the pair $(\bar{q}_j, \bar{x}_j)$ that solves the pair of equations:

$$x_j = \frac{(\rho + \delta_j)\,\epsilon_j}{\alpha_j \theta_j (\epsilon_j - 1)} (1 + q_j);$$

$$x_j = \frac{\phi_j - \frac{\rho + \delta_j}{\alpha_j}}{\frac{1}{\epsilon_j} - \frac{\theta_j(\epsilon_j - 1)}{\epsilon_j(1+q_j)} - \beta(\rho + \delta_j)}.$$

We thus obtain:

$$\bar{x}_j = \frac{\epsilon_j \phi_j}{1 - \epsilon_j \beta (\rho + \delta_j)};$$

$$\bar{q}_j = \frac{\phi_j}{1 - \epsilon_j \beta (\rho + \delta_j)} \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\rho + \delta_j} - 1.$$

The solution $\bar{x}_j$ is the steady state with no firm innovation of the model with no transportation cost (e.g., Peretto 2025).

The solution $\bar{q}_j$ says that the model with per-unit transportation cost adds to the theory two novel properties. First, the per-unit transportation cost causes stagnation because cost reduction is no longer an economically self-sustaining process. Second, the model exhibits an endogenous cost structure whereby in steady state the unit cost of manufacturing converges to

$$\bar{Z}_j^{-\theta_j} = \frac{\eta_j}{\bar{q}_j}.$$

Then, the iceberg transportation cost is

$$\tau_j \bar{Z}_j^{-\theta_j} = \tau_j \frac{\eta_j}{\bar{q}_j}$$

and accounts for the fraction

$$\frac{\tau_j \frac{\eta_j}{\bar{q}_j}}{\tau_j \frac{\eta_j}{\bar{q}_j} + (1+\tau_j)\,\eta_j} = \frac{\tau_j}{\tau_j + (1+\tau_j)\,\bar{q}_j}$$

of the unit expenditure on transportation. In turn, transportation accounts for the fraction

$$\frac{\tau_j \frac{\eta_j}{\bar{q}_j} + (1+\tau_j)\,\eta_j}{\tau_j \frac{\eta_j}{\bar{q}_j} + (1+\tau_j)\,\eta_j + \frac{\eta_j}{\bar{q}_j}} = \frac{\tau_j + (1+\tau_j)\,\bar{q}_j}{(1+\tau_j)\,(1+\bar{q}_j)}$$

of the total unit cost of producing and delivering the good to the customer.

The path-dependent trajectories obey the equation

$$\frac{\dot{x}_j}{x_j} = \rho + \delta_j - \frac{1}{\beta x_j}\left[\frac{x_j}{\epsilon_j} - \phi_j - \frac{\theta_j\,(\epsilon_j - 1)}{\epsilon_j}\frac{x_j}{1+q_j\,(0)} + \frac{\rho+\delta_j}{\alpha_j}\right],$$

where $q_j\,(0) = \eta_j\,[Z_j\,(0)]^{\theta_j}$. These dynamics yield that firm size converges to

$$x_j\,(q_j\,(0)) = \frac{\left(\phi_j - \frac{\rho+\delta_j}{\alpha_j}\right)[1+q_j\,(0)]}{\left[\frac{1}{\epsilon_j} - \beta\,(\rho+\delta_j)\right][1+q_j\,(0)] - \frac{\theta_j(\epsilon_j-1)}{\epsilon_j}}.$$

The other properties of these trajectories are similar to those discussed above for the trajectories that converge to $(\bar{q}_j, \bar{x}_j)$.

4.2 Industry-specific TFP

In industry j, the price p_{G_j} is equal to the price index of the intermediates it uses. Hence,

$$\begin{aligned} p_{G_j} &= N_j^{-\chi_j}\left(\frac{1}{N_j}\int_0^{N_j} p_{X_{ij}}^{1-\epsilon_j}\,di\right)^{\frac{1}{1-\epsilon_j}} \\ &= \frac{\epsilon_j}{\epsilon_j - 1}\,(1+\tau_j)\,N_j^{-\chi_j} Z_j^{-\theta_j}\left(1+\eta_j Z_j^{\theta_j}\right). \end{aligned}$$

Accordingly, the industry's output is

$$G_j = \frac{Y_j}{p_{G_j}} = \frac{\varphi_j Y}{\frac{\epsilon_j}{\epsilon_j - 1}(1+\tau_j)} \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}}.$$

Noting that industry employment is $L_j = \varphi_j L$, industry output per worker is

$$\frac{G_j}{L_j} = \frac{\varphi_j Y}{\varphi_j L \frac{\epsilon_j}{\epsilon_j - 1}(1+\tau_j)} \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}} = \frac{y}{\frac{\epsilon_j}{\epsilon_j - 1}(1+\tau_j)} \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}},$$

We define industry TFP as

$$T_j \equiv \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}}.$$

Accordingly, the industry growth rate of output per worker (industry growth for brevity) is

$$g_j = \frac{\dot{y}}{y} + \frac{\dot{T}_j}{T_j} = \chi_j n_j + \theta_j z_j - \frac{\eta_j \theta_j Z_j^{\theta_j}}{1+\eta_j Z_j^{\theta_j}} z_j = \chi_j n_j + \frac{\theta_j z_j}{1+\eta_j Z_j^{\theta_j}}.$$

This equation shows that as long as there is a per-unit transportation cost, endogenous growth driven by cost reduction is not possible.

In steady state, we have

$$g_j^* = \chi_j \lambda,$$

and the model determines only the level of TFP. In logs, we have

$$\log T_j(t) = \theta_j \log Z_j(0) + \chi_j \log N_j(0) + \log\left(\frac{\bar{Z}_j^{\theta_j}}{1+\eta_j \bar{Z}_j^{\theta_j}}\right) + \lambda t.$$

The key result here is that the only driver of TFP growth is the growth of the population via the weak scale effect. To highlight the role of the transportation parameters, we write the solution in the form

$$\log T_j(t) = \theta_j \log Z_j(0) + \chi_j \log N_j(0) + \log\left(\frac{\bar{q}_j}{1+\bar{q}_j}\right) - \log \eta_j + \lambda t.$$

Recalling that $\bar{q}_j$ does not depend on transportation parameters, this solution says that the per-unit transportation cost has a one-to-one negative effect on the steady-state path of the industry's TFP. Notice that the transportation parameter τ_j is absent from this expression.

The reason is that the iceberg component of the transportation cost is proportional to the unit production cost and is thus fully absorbed by the firm's gross profit x_j/ϵ_j via the constant markup. In other words, that component of the transportation cost does not affect firm size and thus has no effect on the dynamics of TFP.

4.3 Aggregation and welfare

The price of the consumption good is the price index of the components that it embodies,

$$p_G = \prod_{j=1}^{J} \left(\frac{p_{G_j}}{\varphi_j} \right)^{\varphi_j} = \xi \times \prod_{j=1}^{J} T_j^{-\varphi_j}, \tag{26}$$

where we define the aggregate index of static cost drivers,

$$\xi \equiv \prod_{j=1}^{J} \left(\frac{\frac{\epsilon_j}{\epsilon_j - 1} (1 + \tau_j)}{\varphi_j} \right)^{\varphi_j}.$$

We then use the price index (26) to write GDP per capita

$$\frac{G}{L} = \frac{y}{p_G} = \frac{y}{\xi} \cdot \underbrace{\prod_{j=1}^{J} T_j^{\varphi_j}}_{\text{aggregate TFP} \equiv T}.$$

Accordingly, aggregate growth is

$$g = \frac{\dot{y}}{y} + \frac{\dot{T}_j}{T_j} = \frac{\dot{y}}{y} + \sum_{j=1}^{J} \varphi_j \frac{\dot{T}_j}{T_j} = \frac{\dot{y}}{y} + \sum_{j=1}^{J} \varphi_j \left(\chi_j n_j + \frac{\theta_j z_j}{1 + \eta_j Z_j^{\theta_j}} \right).$$

In steady state

$$g^* = \left(\frac{\dot{T}}{T} \right)^* = \sum_{j=1}^{J} \varphi_j^* \left(\frac{\dot{T}_j}{T_j} \right)^* = \sum_{j=1}^{J} \varphi_j \chi_j \lambda.$$

Thus, aggregate TFP growth is the weighted sum of industry growth, where the weights are the industry shares of GDP.

These results let us write individual utility as

$$
\begin{aligned}
u &= \log\left(\frac{y}{\xi}\right) + \sum_{j=1}^{J} \varphi_j \log T_j \\
&= \log y - \sum_{j=1}^{J} \varphi_j \log\left(\frac{\frac{\epsilon_j}{\epsilon_j - 1}}{\varphi_j}\right) - \sum_{j=1}^{J} \varphi_j \log\left(1+\tau_j\right) + \sum_{j=1}^{J} \varphi_j \log T_j .
\end{aligned}
$$

The second line highlights the role of the transportation parameter τ_j. It has a one-for-one negative effect on flow utility because it adds to the cost of delivering to the customers one unit of the differentiated good j. This is another nice aggregation result. The formula contains only objects that are in principle observable. For example, we can rewrite the first term as $\log\left(y/p_G\right) + \log\left(p_G/c\right)$, where the first term of this decomposition is the log of GDP per capita and the second term is the log of the fraction of the economy's price index accounted for by the static drivers of the cost of producing one unit of GDP. In the remainder of the paper we exploit this structure to study how the interdependence of manufacturing and transportation determines the welfare achieved by the representative household in market equilibrium.

5 Innovation in transportation

In this section we study the model with innovation in transportation. We set $\sigma_j > 0$ for all j so that the endogenous outcome $L_{S_j} > 0$ for all j can eventually occur in all industries.

5.1 Industry-specific dynamics

Recall the definition and interpretation of the variable $q_j = \eta_j S_j^{-\sigma_j} Z_j^{\theta_j}$. In general equilibrium, we obtain the following structure. Firm manufacturing innovation is

$$
z_j = \alpha_j L_{Z_j} = \frac{\alpha_j \theta_j \left(\epsilon_j - 1\right)}{\epsilon_j} \frac{1}{1+q_j} x_j - \rho - \delta_j, \tag{27}
$$

where $L_{Z_j} > 0$ for

$$
x_j > x_{Z_j}\left(q_j\right) \equiv \frac{\epsilon_j \left(\rho + \delta_j\right)}{\alpha_j \sigma_j (\epsilon_j - 1)} \left(1 + q_j\right).
$$

Firm transportation innovation is

$$s_j \equiv \frac{\dot{S}_j}{S_j} = \varsigma_j L_{S_j} = \frac{\varsigma_j \sigma_j (\epsilon_j - 1)}{\epsilon_j} \frac{q_j}{1+q_j} x_j - \rho - \delta_j, \tag{28}$$

where $L_{S_j} > 0$ for

$$x_j > x_{S_j}(q_j) \equiv \frac{\epsilon_j (\rho + \delta_j)}{\varsigma_j \sigma_j (\epsilon_j - 1)} \frac{1+q_j}{q_j}.$$

We now use these equations to construct the dynamical system.

We take log and time-derivative of $q_j = \eta_j S_j^{-\sigma_j} Z_j^{\theta_j}$ to write

$$\begin{aligned} \frac{\dot{q}_j}{q_j} &= \theta_j z_j - \sigma_j s_j \\ &= \frac{\epsilon_j - 1}{\epsilon_j} \frac{\alpha_j \theta_j^2 - \varsigma_j \sigma_j^2 q_j}{1+q_j} x_j - (\theta_j - \sigma_j)(\rho + \delta_j). \end{aligned}$$

We then use equations (27)-(28) and the expression for the rate of return to entry,

$$\rho = \frac{1}{\beta x_j} \left(\frac{x_j}{\epsilon_j} - \phi_j - L_{Z_j} - L_{S_j} \right) + \frac{\dot{x}_j}{x_j} - \delta_j,$$

to write

$$\frac{\dot{x}_j}{x_j} = \rho + \delta_j - \frac{1}{\beta} \left[\frac{1}{\epsilon_j} - \frac{\sigma_j q_j - \theta_j}{1+q_j} \frac{\epsilon_j - 1}{\epsilon_j} - \frac{\phi_j - (\rho + \delta_j)\left(\frac{1}{\alpha_j} + \frac{1}{\varsigma_j}\right)}{x_j} \right]. \tag{29}$$

We thus have a reasonably simple industry-specific dynamical system in (q_j, x_j) space (see the appendix for the details). Figure 2 illustrates the dynamics.

The steady state $\left(q_j^*, x_j^*\right)$ is the solution of the pair of equations:

$$x_j = \frac{(\theta_j - \sigma_j)(\rho + \delta_j)(1+q_j)}{\frac{\epsilon_j - 1}{\epsilon_j}\left(\alpha_j \theta_j^2 - \varsigma_j \sigma_j^2 q_j\right)};$$

$$x_j = \frac{\left[\phi_j - (\rho + \delta_j)\left(\frac{1}{\alpha_j} + \frac{1}{\varsigma_j}\right)\right](1+q_j)}{\frac{1}{\epsilon_j} - \beta(\rho + \delta_j) + \frac{\epsilon_j - 1}{\epsilon_j}\theta_j + \left[\frac{1}{\epsilon_j} - \beta(\rho + \delta_j) - \frac{\epsilon_j - 1}{\epsilon_j}\sigma_j\right] q_j}.$$

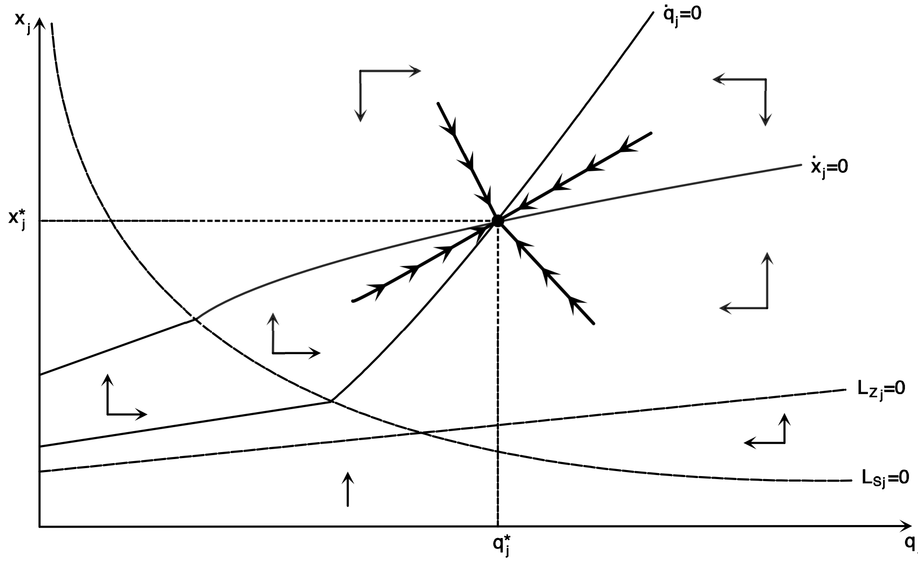

Figure 2: Equilibrium dynamics with innovation in transportation.

Combining them, we obtain:

$$x_j^* = \frac{(\theta_j - \sigma_j)(\rho + \delta_j)\left(1 + q_j^*\right)}{\frac{\epsilon_j - 1}{\epsilon_j}\left(\alpha_j \theta_j^2 - \varsigma_j \sigma_j^2 q_j^*\right)}; \tag{30}$$

$$q_j^* = \frac{\frac{\left[\phi_j - (\rho+\delta_j)\left(\frac{1}{\alpha_j} + \frac{1}{\varsigma_j}\right)\right]\frac{\epsilon_j - 1}{\epsilon_j}\alpha_j \theta_j^2}{(\theta_j - \sigma_j)(\rho + \delta_j)} - \frac{1}{\epsilon_j} + \beta(\rho + \delta_j) - \frac{\epsilon_j - 1}{\epsilon_j}\theta_j}{\frac{\left[\phi_j - (\rho+\delta_j)\left(\frac{1}{\alpha_j} + \frac{1}{\varsigma_j}\right)\right]\frac{\epsilon_j - 1}{\epsilon_j}\varsigma_j \sigma_j^2}{(\theta_j - \sigma_j)(\rho + \delta_j)} + \frac{1}{\epsilon_j} - \beta(\rho + \delta_j) - \frac{\epsilon_j - 1}{\epsilon_j}\sigma_j}. \tag{31}$$

This is the industry's new labor productivity ratio relative to the case $\sigma_j = 0$ in which firms do not invest to improve the productivity of labor in transportation. It is worth stressing the absence of the parameter η_j from this solution. The reason is that changes in η_j are fully absorbed by the adjustment of the two stocks Z_j and S_j, which in equilibrium comove to produce the constant value q_j^* in equation (31) that does not depend on η_j. The insight follows form the definition of the variable q_j: it is the ratio between labor productivity in manufacturing to labor productivity in shipping; the no-arbitrage equilibrium of the market says that this ratio must be constant in steady state at a value determined by the parameters governing the rates of return to innovation in manufacturing and in transportation and the rate of return to entry.

5.2 Industry-specific cost and employment structure

The production cost, the iceberg transportation cost and the per-unit transportation cost go to zero asymptotically but remain in constant ratios. In particular, the iceberg transportation cost accounts for the fraction

$$\frac{\tau_j Z_j^{-\theta_j}}{\tau_j Z_j^{-\theta_j} + (1+\tau_j)\,\eta_j S_j^{-\sigma_j}} = \frac{\tau_j}{\tau_j + (1+\tau_j)\,q_j^*}$$

of the unit expenditure on transportation. In turn, transportation accounts for the fraction

$$\frac{\tau_j Z_j^{-\theta_j} + (1+\tau_j)\,\eta_j S_j^{-\sigma_j}}{\tau_j Z_j^{-\theta_j} + (1+\tau_j)\,\eta_j S_j^{-\sigma_j} + Z_j^{-\theta_j}} = \frac{\tau_j + (1+\tau_j)\,q_j^*}{(1+\tau_j)\left(1+q_j^*\right)}$$

of the total unit cost of delivering the good to the customer. Note that in this case as well the steady-state value of the variable q does not depend on the transportation cost parameters τ_j and η_j. This is another manifestation of the model's scale invariance. However, q^* depends on the parameters of the transportation innovation technology, σ_j and ς_j, via the no-arbitrage principle at the heart of the model's market equilibrium.

We also have the employment structure:

$$L_{X_j}^* = \frac{\epsilon_j - 1}{\epsilon_j}\frac{x_j^*}{(1+\tau_j)\left(1+q_j^*\right)} + \phi_j;$$

$$L_{D_j}^* = \frac{\epsilon_j - 1}{\epsilon_j}\frac{\tau_j + (1+\tau_j)\,q_j^*}{w\,(1+\tau_j)\left(1+q_j^*\right)}x_j^*;$$

$$L_{Z_j}^* = \frac{\theta_j\,(\epsilon_j - 1)}{\epsilon_j}\frac{x_j^*}{1+q_j^*} - \frac{\rho+\delta_j}{\alpha_j};$$

$$L_{S_j}^* = \frac{\varsigma_j\sigma_j(\epsilon_j - 1)}{\epsilon_j}\frac{q_j^* x_j^*}{1+q_j^*} - \frac{\rho+\delta_j}{\varsigma_j};$$

$$L_{N_j}^* = (\lambda+\delta)\,\beta\varphi_j y^* L.$$

Note that while the pair $\left(q_j^*, x_j^*\right)$ does not depend on the iceberg component of the transportation cost, τ_j, the industry-specific allocation of labor to making things, $L_{X_j}^*$, and to shipping things, $L_{D_j}^*$, does depend on it simply because the iceberg cost contributes to the wedge between the cost of producing the good and the cost of delivering the good to the customer, where we think of delivering as making the good plus shipping the good. Another way to see this property is that while τ_j does not affect the industry's innovation and market structure dynamics, it obviously affects the price that the customer pays for the good. This implies that τ_j affects the utility that economic activity generates for the representative household. We now turn to this aspect of the model.

5.3 Industry-specific TFP growth

In industry j, the price p_{G_j} is equal to the price index of the intermediates it uses. Hence,

$$\begin{aligned} p_{G_j} &= N_j^{-\chi_j}\left(\frac{1}{N_j}\int_0^{N_j} p_{X_{ij}}^{1-\epsilon_j}di\right)^{\frac{1}{1-\epsilon_j}} \\ &= \frac{\epsilon_j}{\epsilon_j-1}\left(1+\tau_j\right)N_j^{-\chi_j}Z_j^{-\theta_j}\left(1+S_j^{-\sigma_j}Z_j^{\theta_j}\right) \\ &= \frac{\epsilon_j}{\epsilon_j-1}\left(1+\tau_j\right)N_j^{-\chi_j}Z_j^{-\theta_j}\left(1+q_j\right). \end{aligned}$$

Accordingly, the industry's output is

$$G_j = \frac{Y_j}{p_{G_j}} = \frac{\varphi_j Y}{\frac{\epsilon_j}{\epsilon_j-1}\left(1+\tau_j\right)}\frac{Z_j^{\theta_j}N_j^{\chi_j}}{1+q_j}.$$

Noting that industry employment is $L_j = \varphi_j L$, industry output per worker is

$$\frac{G_j}{L_j} = \frac{\varphi_j Y}{\varphi_j L\frac{\epsilon_j}{\epsilon_j-1}\left(1+\tau_j\right)}\frac{Z_j^{\theta_j}N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}} = \frac{y}{\frac{\epsilon_j}{\epsilon_j-1}\left(1+\tau_j\right)}\frac{Z_j^{\theta_j}N_j^{\chi_j}}{1+\eta_j S_j^{-\sigma_j}Z_j^{\theta_j}},$$

where $Z_j^{\theta_j}$ is manufacturing productivity and $S_j^{\sigma_j}/\eta_j$ is transportation productivity. We define industry TFP as

$$T_j \equiv \frac{Z_j^{\theta_j}N_j^{\chi_j}}{1+q_j} = \frac{Z_j^{\theta_j}N_j^{\chi_j}S_j^{\sigma_j}}{S_j^{\sigma_j}+\eta_j Z_j^{\theta_j}},$$

Note that TFP is no longer a power function of manufacturing knowledge, Z_j, and is not a power function of transportation knowledge, S_j. The reason is that the per-unit transportation cost causes an interaction between production and distribution that results in industry TFP being concave in manufacturing productivity and concave in transportation productivity separately. Industry TFP, however, is homogeneous of degree one (i.e., linear) in manufacturing productivity and transportation productivity jointly. This property yields endogenous growth. In particular, the industry growth rate of output per worker (industry growth for brevity) is

$$g_j = \frac{\dot{y}}{y} + \frac{\dot{T}_j}{T_j} = \chi_j n_j + \theta_j z_j - \frac{q_j}{1+q_j}\frac{\dot{q}_j}{q_j}.$$

In steady state, $\dot{q}_j = 0$ and therefore

$$g_j^* = \left(\frac{\dot{T}_j}{T_j}\right)^* = \chi_j \lambda + \theta_j z_j^*,$$

where:

$$z_j^* = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j^*}{1+q_j^*} - \rho - \delta_j;$$

$$s_j^* = \frac{\theta_j}{\sigma_j} z_j^*.$$

This equation says that in this model with heterogeneous industries the theory's scale invariance property holds industry by industry. Each industry-specific growth rate, g_j^*, is independent of the population scale factor, Λ, and of the industry employment share, φ_j, and depends on the population growth rate, λ, only through the Romer component, $\chi_j \lambda$, due to the weak scale effect. It is also independent of the transportation cost parameters τ_j and η_j.

5.4 Aggregate growth and welfare

Aggregation is the same as in the model with $\sigma_j = 0$ for all j, up to the dynamics of TFP. Thus, GDP per capita is again

$$\frac{G}{L} = \frac{y}{p_G} = \frac{y}{\xi} \cdot \underbrace{\prod_{j=1}^{J} T_j^{\varphi_j}}_{\text{aggregate TFP} \equiv T},$$

where ξ is the composite parameter defined in Section 4 and

$$T_j \equiv \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+q_j}.$$

Aggregate growth is

$$g = \frac{\dot{y}}{y} + \frac{\dot{T}}{T} = \frac{\dot{y}}{y} + \sum_{j=1}^{J} \varphi_j \left(\frac{\dot{T}_j}{T_j}\right)^* = \frac{\dot{y}}{y} + \sum_{j=1}^{J} \varphi_j \left(\chi_j n_j + \theta_j z_j - \frac{q_j}{1+q_j}\frac{\dot{q}_j}{q_j}\right).$$

In steady state, $\dot{q}_j = 0$ for all j so that

$$g^* = \left(\frac{\dot{T}}{T}\right)^* = \sum_{j=1}^{J} \varphi_j \left(\frac{\dot{T}_j}{T_j}\right)^* = \sum_{j=1}^{J} \varphi_j \left(\chi_j \lambda + \theta_j z_j^*\right).$$

Thus, aggregate TFP growth is the weighted sum of industry growth, where the weights are the industry shares of GDP.

These results let us write individual utility as

$$u = \log\left(\frac{y}{\xi}\right) + \sum_{j=1}^{J} \varphi_j \log T_j.$$

This is another nice aggregation result. In the appendix we show that linearizing the dynamical system around the steady state we can compute

$$\frac{U^*}{\Lambda} = \frac{\log\left(\frac{y^*}{\xi}\right) + \varkappa}{\rho - \lambda} + \sum_{j=1}^{J} \frac{\varphi_j g_j^*}{(\rho - \lambda)^2} + \sum_{j=1}^{J} \frac{\varphi_j \Upsilon_j}{\rho - \lambda},$$

where:

$$\varkappa \equiv \chi_j \log \varphi_j + \chi_j \log y^* + \chi_j \log \Lambda;$$

$$\Upsilon_j \equiv \left[\theta_j - \frac{\theta_j + \chi_j}{\rho - \lambda + \nu_{x_j}}\right] \Delta_{x_j} - \left[\theta_j - \frac{\theta_j + \chi_j}{\rho - \lambda + \nu_{q_j}}\right] \Delta_{q_j};$$

$$\Delta_{x_j} \equiv x_j(0) - x_j^*;$$

$$\Delta_{q_j} \equiv q_j(0) - q_j^*.$$

This expression describes the welfare per capita produced by the aggregation of the J industry-specific transition paths, each one going from the initial point $(q_j(0), x_j(0))$ to the steady state (q_j^*, x_j^*). In particular, in the expression for the composite factor Υ_j, the terms ν_{q_j} and ν_{x_j} are the eigenvalues of the linearized dynamical system attached to, respectively, q_j and x_j, and the terms Δ_{q_j} and Δ_{x_j} are the initial gaps of the variables q_j and x_j from their steady states; see the appendix for the derivation and the details.

6 Comparative statics and dynamics

In this section we study the dynamic effects of changes in parameters. We focus on the parameters of the transportation technology. We also consider the fixed operating cost because the recent literature on business dynamism and the productivity slowdown documents that it plays a central role in explaining the trends in the data (see, e.g., xxx, xxx, and xxx). In our model, it plays the same central role with the added insight that it also works through its effects on the dynamic interdependence of manufacturing and shipping.

6.1 Steady-state comparative statics

We start from the innovation rates:

$$z_j = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{1}{1 + q_j} x_j - \rho - \delta_j;$$

$$s_j = \frac{\varsigma_j \sigma_j (\epsilon_j - 1)}{\epsilon_j} \frac{q_j}{1 + q_j} x_j - \rho - \delta_j.$$

We use the fact that $\theta_j z_j = \sigma_j s_j$ and the equations above to write the $\dot{x}_j = 0$ equation

$$\rho = \frac{1}{\beta x_j} \left(\frac{x_j}{\epsilon_j} - \phi_j - L_{Z_j} - L_{S_j} \right) - \delta_j$$

in the form

$$\phi_j + \left(\frac{1}{\alpha_j} + \frac{\theta_j}{\varsigma_j \sigma_j} \right) z_j = \left[\frac{1}{\epsilon_j} - \beta (\rho + \delta_j) \right] x_j.$$

Next, we use the manufacturing innovation rate to write

$$\frac{z_j + \rho + \delta_j}{\frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j (1 + q_j)}} = x_j.$$

We substitute this expression in the equation above to obtain the function

$$q_j = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{1 - \epsilon_j \beta (\rho + \delta_j)} \frac{\phi_j + \left(\frac{1}{\alpha_j} + \frac{\theta_j}{\varsigma_j \sigma_j} \right) z_j}{z_j + \rho + \delta_j} - 1.$$

This function starts at

$$q_j(0) = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{1 - \epsilon_j \beta (\rho + \delta_j)} \frac{\phi_j}{\rho + \delta_j} - 1$$

and has the horizontal asymptote

$$\lim_{z_j \to \infty} q_j = \frac{\theta_j \left(\epsilon_j - 1\right)}{1 - \epsilon_j \beta \left(\rho + \delta_j\right)} \left(1 + \frac{\alpha_j \theta_j}{\varsigma_j \sigma_j}\right) - 1.$$

For ϕ_j large, the function is decreasing. Since this function comes from the entry process, we call it the free-entry locus.

Now we take the ratio of the innovation rates to write

$$q_j = \frac{\theta_j \alpha_j}{\varsigma_j \sigma_j} \frac{\frac{\theta_j}{\sigma_j} z_j + \rho + \delta_j}{z_j + \rho + \delta_j}.$$

This function starts at

$$q_j \left(0\right) = \frac{\alpha_j \theta_j}{\varsigma_j \sigma_j}$$

and has the horizontal asymptote

$$\lim_{\hat{Z}_j \to \infty} q_j = \frac{\alpha_j \theta_j^2}{\varsigma_j \sigma_j^2}.$$

For $\theta_j > \sigma_j$ the function is increasing. Since this function comes from the definition of the variable q_j, we call it the cost-structure locus.

With this system we can study the effects of parameters in (z_j, q_j) space; see Figure 3. We study the effects of an increase in ϕ_j in red and the effects of an increase in ς_j in blue. Notably absent from this steady-state solution is η_j. We discuss its role below.

The increase in ϕ_j shifts up the free-entry locus and does not move the cost-structure locus. Therefore, both q_j^* and z_j^* rise. Equation (27) then says that x_j^* rises as well. The industry growth rate is $g_j^* = \chi_j \lambda + \theta_j z_j^*$ and thus rises.

The increase in ς_j shifts down both the free-entry locus and the cost-structure locus. Therefore, both q_j^* and z_j^* fall. Equation (27) then says that x_j^* falls as well. The industry growth rate is $g_j^* = \chi_j \lambda + \theta_j z_j^*$ and thus falls. This reflects the reallocation of R&D effort from manufacturing innovation to transportation innovation.

6.2 Comparative dynamics, η_j

The effect of η_j is interesting and reveals something important about the role of transportation in the growth process precisely because η_j does not enter the determination of the steady state in Figure 3. Let η_j^o be the old value and $\eta_j < \eta_j^o$ be the new value. At time $t = 0$, we

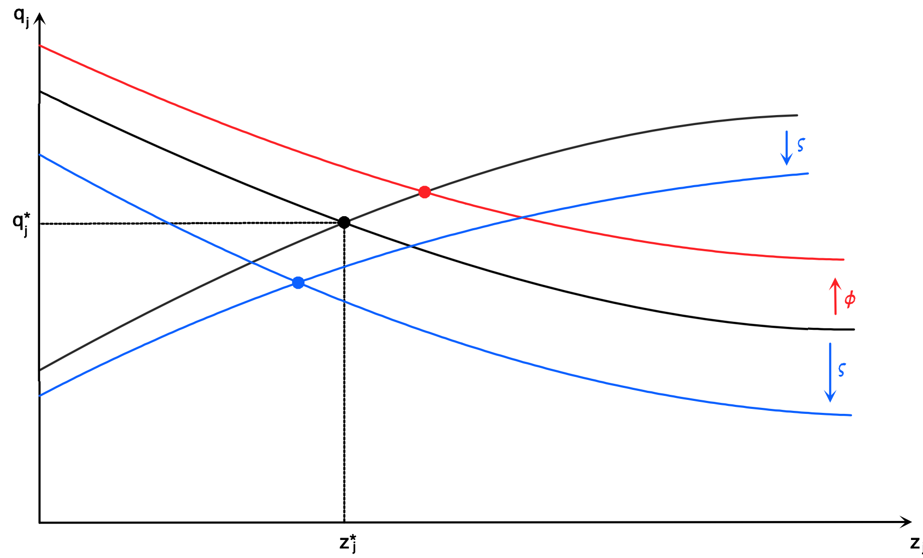

Figure 3: Steady state growth and transportation cost structure, with increase in ϕ_j in red and increase in ς_j in blue.

have a displacement of q_j from q_j^* to the initial condition

$$q_j(0) = \eta_j \cdot [S_j(0)]^{-\sigma_j} [Z_j(0)]^{\theta_j} < q_j^*.$$

By scale invariance, x_j^* does not change. Also, y^* does not change. Therefore, x_i remains at x_j^* at time $t = 0$. The industry then starts a transition from the point $\left(q_j(0), x_j^*\right)$ and returns to the steady state $\left(q_j^*, x_j^*\right)$. Figure 4 illustrates the process. Throughout the process, since q_j grows, we have $\theta_j z_j > \sigma_j s_j$, which means that labor productivity in manufacturing grows faster than labor productivity in transportation.

Industry growth at time $t = 0$ is

$$g_j(0) - g_j^* = \underbrace{\chi_j [n_j(0) - \lambda]}_{\text{positive}} + \underbrace{\theta_j \left[\frac{z_j(0)}{1 + q_j(0)} - z_j^*\right]}_{\text{ambiguous}} + \underbrace{\frac{q_j(0)}{1 + q_j(0)}}_{\text{positive}} \cdot \underbrace{\sigma_j s_j(0)}_{\text{positive}}.$$

This suggests that a sufficient condition for a positive initial jump is

$$\frac{z_j(0)}{1 + q_j(0)} \geq z_j^* \Rightarrow \frac{\frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j^*}{1 + q_j(0)} - \rho - \delta_j}{1 + q_j(0)} \geq z_j^*, \tag{32}$$

where $q_j(0) = \eta_j [S_j(0)]^{-\sigma_j} [Z_j(0)]^{\theta_j}$. If this condition holds, the industry growth rate jumps

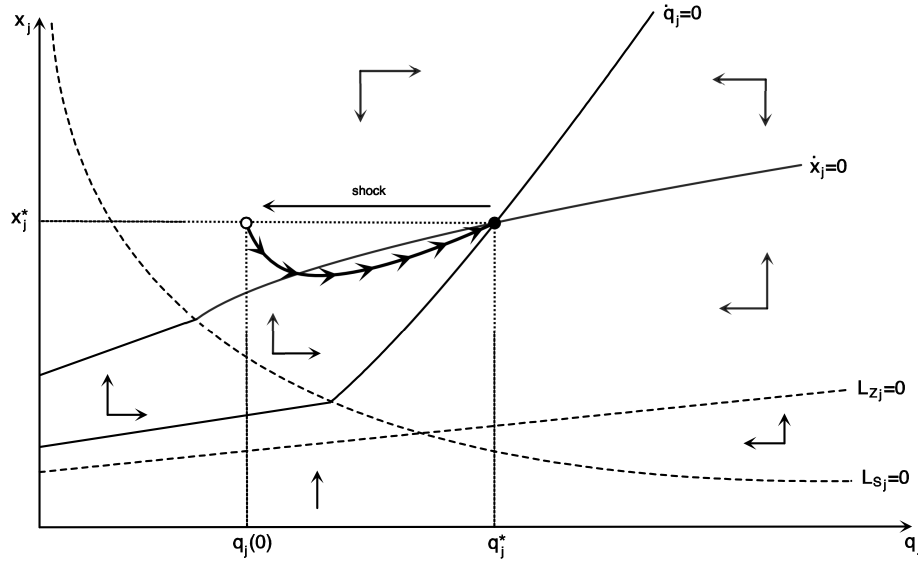

Figure 4: Transitional dynamics in response to an unanticipated, immediate and permanent fall of η_j.

up and then returns gradually to g_j^*, either monotonically or following a hump-shaped path. It's worth stressing this result.

In industries where condition (32) holds, an unanticipated, immediate and permanent fall of the per-unit transportation cost parameter from η_j^o to η_j causes an initial fall of q_j to $q_j(0) < q_j^*$ with unchanged firm size, followed by a transition with q_j returning monotonically to q_j^* and firm size initially falling gradually and then reversing course and returning to x_j^*. The industry growth rate, g_j, jumps up and then returns gradually to g_j^*, either monotonically or following a hump-shaped path.

The components of the growth rate are:

$$z_j = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{1}{1+q_j} x_j - \rho - \delta_j;$$

$$s_j = \frac{\varsigma_j \sigma_j (\epsilon_j - 1)}{\epsilon_j} \frac{q_j}{1+q_j} x_j - \rho - \delta_j;$$

$$\frac{\dot{q}_j}{q_j} = \theta_j z_j - \sigma_j s_j = \frac{\epsilon_j - 1}{\epsilon_j} \frac{\alpha_j \theta_j^2 - \varsigma_j \sigma_j^2 q_j}{1+q_j} x_j - (\theta_j - \sigma_j)(\rho + \delta_j);$$

$$n_j = \lambda + \frac{1}{\epsilon_j \beta} - \frac{\sigma_j q_j - \theta_j}{1+q_j} \frac{\epsilon_j - 1}{\epsilon_j \beta} - \frac{\phi_j - (\rho + \delta_j)\left(\frac{1}{\alpha_j} + \frac{1}{\varsigma_j}\right)}{\beta x_j} - (\rho + \delta_j).$$

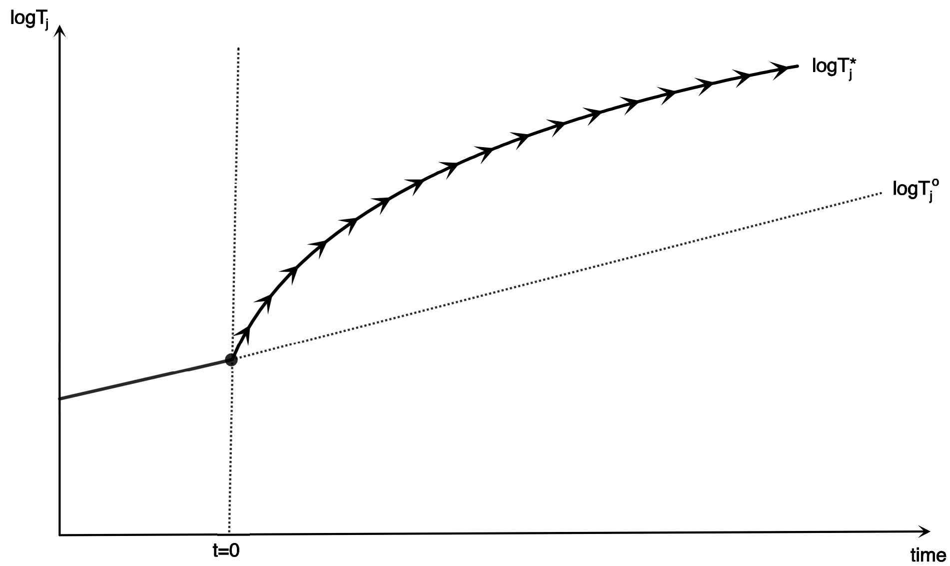

Figure 5: Transitional dynamics in response to an unanticipated, immediate and permanent fall of η_j. The growth acceleration locks in a permanent gain in the level of TFP.

According to these equations, z_j jumps up at $t = 0$ and then returns gradually to z_j^*, s_j jumps down at $t = 0$ and then returns gradually to s_j^*, $\dot{q}_j/q_j$ jumps up at $t = 0$ and then returns gradually to 0, n_j jumps up at $t = 0$ and then returns gradually to λ. Under the sufficient condition (32) discussed above, the net effect is a jump up at time $t = 0$ of the industry growth rate g_j followed by a gradual return to g_j^*.

Figure 5 illustrates the importance of the growth acceleration: it locks in a permanent cumulated gain in the level of TFP despite the fact that the steady-state growth rate does not change with the fall of η_j. Discounting and integrating translates this gain into a welfare gain.

6.3 Comparative dynamics, ς_j

The negative steady-state growth effect of ς_j discussed above can be deceptive. The steady-state growth rate falls but the transition is very important because industry TFP,

$$T_j = \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1 + q_j},$$

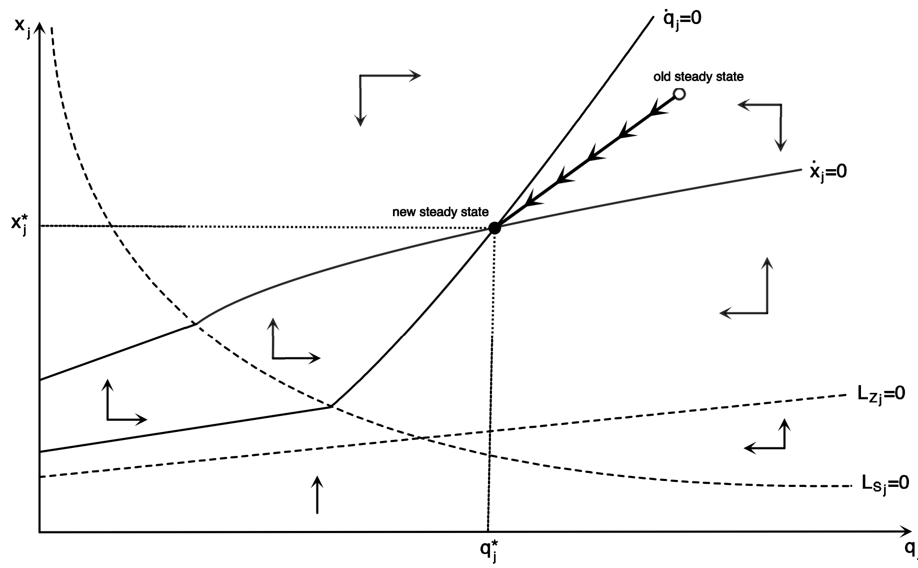

Figure 6: Transitional dynamics in response to an unanticipated, immediate and permanent rise of ς_j.

rises throughout it through the gradual fall of q_j. The transitional growth rate relative to the new steady state is

$$g_j - g_j^* = \underbrace{\chi_j (n_j - \lambda)}_{\text{positive}} + \underbrace{\theta_j \left(z_j - z_j^*\right)}_{\text{ambiguous}} + \underbrace{\left(-\frac{q_j}{1+q_j}\frac{\dot{q}_j}{q_j}\right)}_{\text{positive}}.$$

Figure 6 illustrates the transition for this exercise. Since firm size shrinks, the term n_j rises above the steady-state value λ. Similarly, the last terms is positive since q_j shrinks. A sufficient condition for a growth acceleration is

$$\theta_j \left(z_j - z_j^*\right) = \theta_j \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \left(\frac{x_j}{1+q_j} - \frac{x_j^*}{1+q_j^*}\right) \geq 0$$

throughout the transition.

If this is the case, then we have a dynamic tradeoff between the steady-state effect of ς_j and the transitional effect that yields a cumulative increase in TFP relative to the baseline path. In this exercise, in particular, we have the solution for welfare

$$\frac{U^*}{\Lambda} = \frac{\log\left(\frac{y^*}{c^*}\right) + \varkappa}{\rho - \lambda} + \sum_{j=1}^{J} \frac{\varphi_j g_j^*}{(\rho-\lambda)^2} + \sum_{j=1}^{J} \frac{\varphi_j \Upsilon_j}{\rho - \lambda},$$

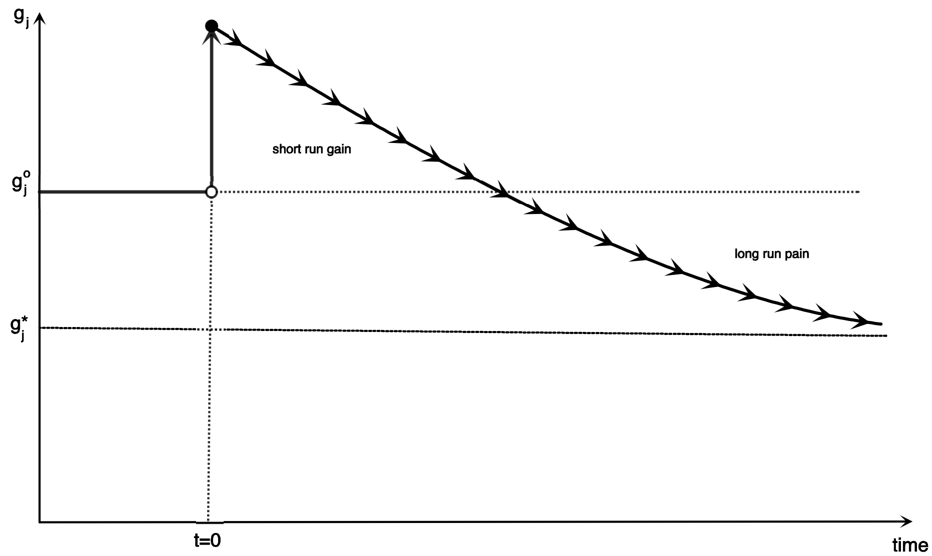

Figure 7: Dynamic tradeoff due to the unanticipated, immediate and permanent increase of ς_j. Cumulating and discounting the initial acceleration and the subsequent deceleration yields the welfare change $U^* - U^o$.

where $\Delta_{x_j} \equiv x_j^o - x_j^*$ and $\Delta_{q_j} \equiv q_j^o - q_j^*$. This expression describes the welfare per capita produced by the aggregation of the J industry-specific transition paths, each one going from the initial steady state $\left(q_j^o, x_j^o\right)$ to the new steady state $\left(q_j^*, x_j^*\right)$.

This exercise says that to gauge the overall effect of a change in the model's environment that causes a reallocation of R&D from one innovation activity to the other, we must account for the transition dynamics — simply looking at the steady-state growth rate can be grossly misleading. In particular, the baseline steady-state path yields welfare per capita

$$\frac{U^o}{\Lambda} = \frac{\log\left(\frac{y^*}{c^*}\right) + \varkappa}{\rho - \lambda} + \sum_{j=1}^{J} \frac{\varphi_j g_j^o}{(\rho - \lambda)^2}.$$

Hence, the effect on welfare per capita of the change in ς_j is

$$\frac{U^* - U^o}{\Lambda} = \frac{1}{\rho - \lambda} \sum_{j=1}^{J} \varphi_j \left[\underbrace{\Upsilon_j}_{\text{positive}} + \underbrace{\frac{g_j^* - g_j^o}{\rho - \lambda}}_{\text{negative}} \right].$$

This is the analytical characterization of the dynamic tradeoff discussed above. This calculation is easily implemented in a quantitative exercise.

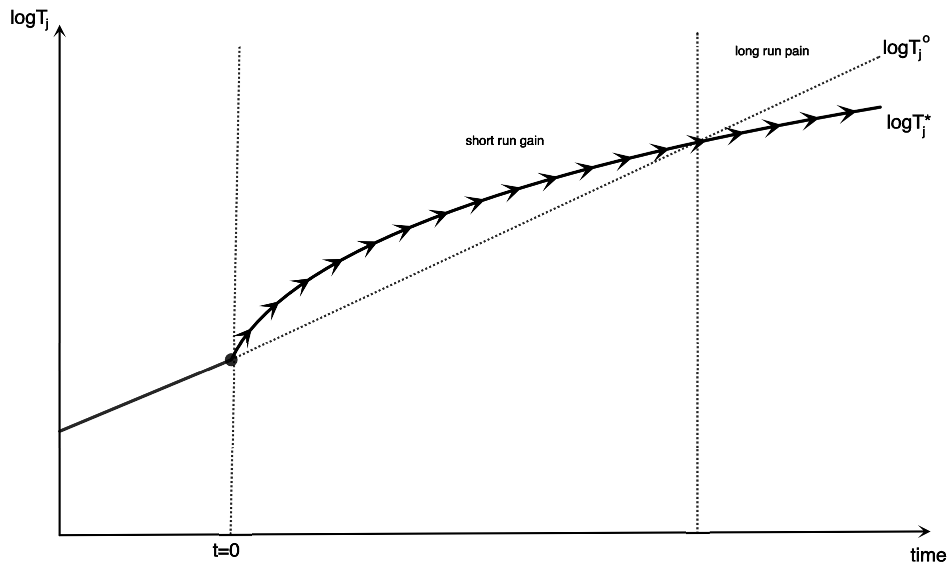

Figure 8: Dynamic tradeoff due to the unanticipated, immediate and permanent increase of ς_j. Cumulated effects on $\log T_j$. Discounting the time path and integrating yields the welfare change $U^* - U^o$.

Figure 7 illustrates the underlying dynamics. The TFP growth rate jumps up and then gradually decreases, going below g_j^o in finite time and then converging to $g_j^* < g_j^o$. The initial acceleration cumulates into a TFP gain relative to the baseline path due to the falling transportation costs. Eventually, however, the reallocation of labor from manufacturing innovation to transportation innovation produces the permanent reduction of the growth rate. The reason is that, as we have seen, the growth rate of labor productivity in transportation is not a driver of steady-state growth. If the transitional effect Υ_j is large enough, despite the steady-state growth deceleration, the economy is better off. Figure 8 plots the time path of the level of the log of TFP to illustrate this property. The welfare calculation above compares two areas of finite size because it multiplies the log of TFP by the subjective discount factor from the household welfare function.

6.4 Summary

This section has illustrated, through a battery of comparative statics and comparative dynamics exercises, the power and usefulness of our model of innovation-led endogenous growth with transportation. When we compare these results to the dynamics generated by the version of the model with no innovation in transportation, we zoom in on the essence of the mechanism driving our results and identify a simple and, to our knowledge, novel insight:

because of the existence of per-unit shipping costs, to understand economic growth we must think in terms of the integrated system "production + transportation" because serving the market consists of making things and shipping things, not just making things. Here things stands for goods and services broadly construed; it ranges from physical objects, like gadgets and people, all the way to intangible services (e.g. legal consultations online) and information (e.g., sending scientific papers through the internet in digital format).

7 Modes of transportation, fuel and regulation

The model has heterogenous industries with industry-specific parameters. In the previous section we have studied the effects of changes in two transportation parameters, η_j and ς_j. In this section we provide a deeper microfoundation of transportation that allows for different modes. Moreover, we extend the production and transportation technologies to allow for a second input that we call fuel (or energy, if one prefers), and model the supply of fuel under two scenarios. Finally, we introduce regulation as a prime mover of the choice of the mix of modes that firms use to ship the things they make to their customers.

7.1 A richer specification of production and transportation

The production and transportation technologies used in the previous sections have only one input, labor. To introduce a second input, fuel, preserving the model's tractability, we replace variable labor with a composite input produced by combining labor with fuel according to an industry-specific technology summarized by the CRS unit cost function $C_j(w, p_f)$, where w is the wage and p_f is the price of fuel. Accordingly, the unit cost of production is $Z_i^{-\theta} C_j(w, p_f)$. Likewise, to introduce different modes of transportation preserving tractability, we define transportation as a technology that produces total distance covered by *one unit* of the firm's good, D_{ij}, by mixing two modes that we call road (r) and water (w). In particular, this transportation technology is summarized by the CRS unit cost function $S_{ij}^{-\sigma_j} C_{D_j}\left(p_{ij}^r, p_{ij}^w\right) D_{ij}$, where p_{ij}^r and p_{ij}^w are the unit prices of road and water transportation.

Road and water transportation, in turn, are produced with labor and fuel according to the technologies $C_j^r(w, f; R^r) = R^r C_j(w, p_f)$ and $C_j^w(w, f; R^w) = R^w C_j(w, p_f)$, where R^r and R^w stand for factors that reduce the productivity of labor and fuel in, respectively, road transportation and water transportation. These factors can be geographical, e.g., properties of the landscape, like whether it is relatively flat or rugged and whether there are abundant natural waterways like rivers, and/or institutional like, for example, regulations.[6] The interpretation of this characterization is that in this economy production and transportation use labor and fuel with the same factor intensity, described by the function $C_j(w, p_f)$, and differ only by the Hicks neutral TFP terms Z_i^{θ}, $S_{ij}^{\sigma_j}$, $1/R^r$ and $1/R^w$.[7]

7.2 Decisions: minimizing the per unit cost of shipping

To characterize the firm's decisions, we exploit the time separability of the value-maximization problem, namely, the property that the decisions concerning the good's price and the use of the variable inputs given their market prices are intratemporal and can be solved for given values of the state variables Z_{ij} and S_{ij}. In particular, we imagine that the firm consists of a production plus investment division and a distribution division that consists of two subdivisions, road and water, each of which charges an internal price for its services. We call these prices p_{ij}^r and p_{ij}^w and note that internal efficiency dictates $p_{ij}^r = R^r C_j(w, p_f)$ and $p_{ij}^w = R^w C_j(w, p_f)$. The distribution division then charges the internal price

$$\begin{aligned} p_{D_{ij}} &= S_{ij}^{-\sigma_j} C_{D_j}\left(p_{ij}^r, p_{ij}^w\right) D_{ij} \\ &= S_{ij}^{-\sigma_j} C_{D_j}\left(R^r C_j(w, p_f), R^w C_j(w, p_f)\right) D_{ij} \\ &= S_{ij}^{-\sigma_j} C_j(w, p_f) C_{D_j}(R^r, R^w) D_{ij}, \end{aligned}$$

which is simply the per unit cost of shipping things ovear a set distance $D_{ij} = 1$, which we normalize to 1 for simplicity and without loss of generality, determined by the cost-minimizing choice of the two modes. With this structure, the firm's total expenditure on shipping is $p_{D_{ij}} X_{ij}$. Therefore, we modify its value-maximization problem to

$$\begin{aligned} CVH_i &= \underbrace{\left[p_{X_{ij}} - (1+\tau_j) Z_i^{-\theta} C_j(w, p_f) - p_{D_{ij}}\right] X_{ij}}_{\text{intratemporal}} - w\phi_j \\ &\quad - wL_{Z_{ij}} - wL_{S_{ij}} + \upsilon_{Z_{ij}} \alpha_j Z_j L_{Z_{ij}} + \upsilon_{S_{ij}} \varsigma_j S_j L_{S_{ij}}, \end{aligned}$$

where the key modifications are the new terms representing the per unit cost of shipping and the price of fuel.

[6] In more ambitious specifications that we leave to future work, R^r and R^w could be linked to the concept of infrastructure modeled as an endogenous variable (e.g., public capital).

[7] We stress that this assumption of equal factor intensity is key to the tractability of the model's dynamics. If we break it, the model becomes too complex to solve with qualitative methods.

In this notation, the efficient choice of modes of transportation, and of the use of labor and fuel in each mode, is subsumed in the per unit cost of shipping $p_{D_{ij}}$. In particular, allocating labor and fuel to road transportation rather than water transportation matters to the firm because (1) the two modes are *not* perfect substitutes, indeed our technology allows for both substitutability and complementarity, and (2) they have different Hicks neutral TFP due to regulation. This property has desirable implications for our purposes. We write the cost shares of the two modes in the form:

$$c_j^r\left(p_{ij}^r, p_{ij}^w\right) \equiv \frac{\partial \log C_{D_j}\left(p_{ij}^r, p_{ij}^w\right)}{\partial \log p_{ij}^r};$$

$$c_j^w\left(p_{ij}^r, p_{ij}^w\right) \equiv \frac{\partial \log C_{D_j}\left(p_{ij}^r, p_{ij}^w\right)}{\partial \log p_{ij}^w}.$$

Note that by definition $c_j^r\left(p_{ij}^r, p_{ij}^w\right)+c_j^w\left(p_{ij}^r, p_{ij}^w\right) = 1$. These shares have the standard properties of cost shares. In particular, they allow for substitutability or complementarity between the two modes. In the first case, we have $\partial c_j^r\left(p_{ij}^r, p_{ij}^w\right)/\partial p_{ij}^r < 0$ with $\partial c_j^r\left(p_{ij}^r, p_{ij}^w\right)/\partial p_{ij}^w > 0$ and $\partial c_j^w\left(p_{ij}^r, p_{ij}^w\right)/\partial p_{ij}^r > 0$. In the second case, we have $\partial c_j^r\left(p_{ij}^r, p_{ij}^w\right)/\partial p_{ij}^r > 0$ with $\partial c_j^r\left(p_{ij}^r, p_{ij}^w\right)/\partial p_{ij}^w < 0$ and $\partial c_j^w\left(p_{ij}^r, p_{ij}^w\right)/\partial p_{ij}^r < 0$. In the case of substitutability, this simple structure captures the notion that regulations that reduce the productivity of labor and/or fuel in one mode of transportation raise its cost and thus reduce that mode's share of use by firms. Moreover, this substitution effect is entirely industry specific.

We concentrate on the intratemporal part of the firm's problem. The price decision is

$$p_{X_{ij}} = \frac{\epsilon_j}{\epsilon_j - 1}\left[\left(1+\tau_j\right) Z_{ij}^{-\theta} C_j\left(w, p_f\right) + p_{D_{ij}}\right],$$

which yields gross profit $p_{X_{ij}} X_{ij}/\epsilon_j$. The firm's expenditures on making and shipping things are:

$$\text{production} = \frac{Z_{ij}^{-\theta_j} C_j\left(w, p_f\right)}{\left(1+\tau_j\right) Z_i^{-\theta} C_j\left(w, p_f\right) + p_{D_{ij}}} \cdot \frac{\epsilon_j - 1}{\epsilon_j} p_{X_{ij}} X_{ij};$$

$$\text{transportation} = \frac{\tau_j Z_{ij}^{-\theta_j} + p_{D_{ij}}}{\left(1+\tau_j\right) Z_i^{-\theta} C_j\left(w, p_f\right) + p_{D_{ij}}} \cdot \frac{\epsilon_j - 1}{\epsilon_j} p_{X_{ij}} X_{ij}.$$

By construction, these expressions add up to one minus the profit rate. This is because the expression before the dot are properly defined shares that add up to one.

An example can be useful to understand better this structure. Let the function $C_{D_j}\left(p_{ij}^r, p_{ij}^w\right)$ be the price index of a CES production function with elasticity of substitution ϖ_j and share parameter ϱ_j, that is,

$$C_{D_j}\left(p_{ij}^r, p_{ij}^w\right) = \left[\left(\varrho_j\right)^{\varpi_j}\left(p_{ij}^r\right)^{1-\varpi_j} + \left(1-\varrho_j\right)^{\varpi_j}\left(p_{ij}^w\right)^{1-\varpi_j}\right]^{\frac{1}{1-\varpi_j}}, \quad \varpi_j > 0,\ 0 < \varrho_j < 1.$$

Moreover, let

$$C_j\left(w, p_f\right) = \vartheta_j w^{\omega_j} p_f^{1-\omega_j}, \quad \vartheta_j \equiv \left(\frac{\omega_j}{1-\omega_j}\right)^{1-\omega_j} + \left(\frac{1-\omega_j}{\omega_j}\right)^{\omega_j}.$$

With these technologies, we have

$$C_{D_j}\left(R^r, R^w\right) = \left[\left(\varrho_j\right)^{\varpi_j}\left(R^r\right)^{1-\varpi_j} + \left(1-\varrho_j\right)^{\varpi_j}\left(R^w\right)^{1-\varpi_j}\right]^{\frac{1}{1-\varpi_j}}$$

and the choice of mode is:

$$c_j^r\left(p_{ij}^r, p_{ij}^w\right) = \frac{\left(p_{ij}^r\right)^{1-\varpi_j}}{\left(p_{ij}^r\right)^{1-\varpi_j} + \left(p_{ij}^w\right)^{1-\varpi_j}} = \frac{1}{1+\left(\frac{R^w}{R^r}\right)^{1-\varpi_j}} \equiv c_j^r\left(R^w/R^r\right);$$

$$c_j^w\left(p_{ij}^r, p_{ij}^w\right) = \frac{\left(p_{ij}^w\right)^{1-\varpi_j}}{\left(p_{ij}^r\right)^{1-\varpi_j} + \left(p_{ij}^w\right)^{1-\varpi_j}} = \frac{\left(\frac{R^w}{R^r}\right)^{1-\varpi_j}}{1+\left(\frac{R^w}{R^r}\right)^{1-\varpi_j}} \equiv c_j^w\left(R^w/R^r\right).$$

This expression says that the road transportation share, c_j^r, is decreasing in R^w/R^r for $\varpi_j < 1$ (complementarity) and increasing in R^w/R^r for $\varpi_j > 1$ (substitutability).

We now use the assumption that the composite input is produced combining labor and fuel according to a Cobb-Douglas technology with labor intensity ω_j and the expression for $p_{D_{ij}}$ to obtain the split of the firm's expenditure on labor and fuel in production and transportation:[8]

$$w\left(L_{X_{ij}} - \phi\right) = \frac{Z_{ij}^{-\theta_j}}{\left(1+\tau_j\right) Z_{ij}^{-\theta} + S_{ij}^{-\sigma_j} C_{D_j}\left(R^r, R^w\right)} \cdot \omega_j \frac{\epsilon_j - 1}{\epsilon_j} p_{X_{ij}} X_{ij}; \tag{33}$$

$$p_f F_{X_{ij}} = \frac{Z_{ij}^{-\theta_j}}{\left(1+\tau_j\right) Z_{ij}^{-\theta} + S_{ij}^{-\sigma_j} C_{D_j}\left(R^r, R^w\right)} \cdot \left(1-\omega_j\right) \frac{\epsilon_j - 1}{\epsilon_j} p_{X_{ij}} X_{ij}; \tag{34}$$

$$w L_{D_{ij}} = \frac{\tau_j Z_{ij}^{-\theta_j} + S_{ij}^{-\sigma_j} C_{D_j}\left(R^r, R^w\right) D_{ij}}{\left(1+\tau_j\right) Z_{ij}^{-\theta} C_j\left(w, p_f\right) + S_{ij}^{-\sigma_j} C_{D_j}\left(w, p_f\right) C_j\left(R^r, R^w\right)} \cdot \omega_j \frac{\epsilon_j - 1}{\epsilon_j} p_{X_{ij}} X_{ij}; \tag{35}$$

[8]The results below do not require the Cobb-Douglas assumption, they would hold with a generic unit cost function. However, the Cobb-Douglas assumption makes the notation and the algebra much simpler.

$$p_f F_{D_{ij}} = \frac{\tau_j Z_{ij}^{-\theta_j} + S_{ij}^{-\sigma_j} C_{D_j}(R^r, R^w) D_{ij}}{(1+\tau_j) Z_{ij}^{-\theta} + S_{ij}^{-\sigma_j} C_{D_j}(R^r, R^w)} \cdot (1-\omega_j) \frac{\epsilon_j - 1}{\epsilon_j} p_{X_{ij}} X_{ij}. \tag{36}$$

The firm's total expenditure on fuel is

$$\begin{aligned} p_f F_{ij} &= \frac{Z_{ij}^{-\theta_j}}{(1+\tau_j) Z_i^{-\theta} + S_{ij}^{-\sigma_j} C_{D_j}(R^r, R^w)} \cdot (1-\omega_j) \frac{\epsilon_j - 1}{\epsilon_j} p_{X_{ij}} X_{ij} \\ &\quad + \frac{\tau_j Z_{ij}^{-\theta_j} + S_{ij}^{-\sigma_j} C_{D_j}(R^r, R^w)}{(1+\tau_j) Z_i^{-\theta} + S_{ij}^{-\sigma_j} C_{D_j}(R^r, R^w)} \cdot (1-\omega_j) \frac{\epsilon_j - 1}{\epsilon_j} p_{X_{ij}} X_{ij} \\ &= (1-\omega_j) \frac{\epsilon_j - 1}{\epsilon_j} p_{X_{ij}} X_{ij}. \end{aligned}$$

This characterization of the firm's behavior is fully in line with the simple model studied in the previous section. In particular, it yields the same expressions for the returns to innovation and entry. This is because the new cost structure of the firm does not change the gross profit rate, it only changes the allocation of the gross profit to the purchase of the inputs used in production and transportation.

7.3 Aggregation and general equilibrium

The expenditure on fuel of industry j is

$$p_f F_j = (1-\omega_j) \frac{\epsilon_j - 1}{\epsilon_j} Y_j.$$

Aggregate expenditure on fuel then is

$$p_f F = \kappa Y, \quad \kappa \equiv \sum_{j=1}^{J} (1-\omega_j) \frac{\epsilon_j - 1}{\epsilon_j} \varphi_j < 1.$$

The household budget in this richer model is

$$\dot{A} = rA + wL + p_f F - Y = rA + wL + \kappa Y - Y,$$

where the only new term is the household's income from selling fuel, $p_f F$. This expression gives us the property highlighted in the analysis of the simple model, namely that the intratemporal equilibrium features constant expenditure per capita,

$$y^* = \frac{1}{1 + \kappa - (\rho - \lambda)\beta},$$

and constant interest rate, $r = \rho$.

Given this intratemporal equilibrium, the value added of this richer characterization of transportation with respect to the simpler model that we studied in the previous sections is that it microfounds the industry-specific parameter η_j. In other words, recalling that we normalize $D_{ij} = 1$, that the industry equilibrium is symmetric, and that in general equilibrium we set $w = 1$, we replace the *exogenous* parameter η_j with the *endogenous* variable $C_j(w, p_f) C_{D_j}(R^r, R^w)$. The value added of this formulation thus is that the firm must ship each unit of its good over a set distance that we normalize to one for simplicity and without loss of generality. Here, however, rather than being an exogenous constant, the cost of shipping one unit of the good is the solution of the cost minimization problem above, which says that the most efficient way to ship the good is a combination of road and water transportation that costs $p_{D_{ij}}$. The shares of the two modes are $c_j^r(R^w/R^r)$ and $c_j^w(R^w/R^r)$, which then yield the pattern of input use (33)-(36).

This extension adds insight to the model but does not change the fundamental property that we uncovered by studying the simpler version that treats η_j as a exogenous parameter. The property is that the scaling constant η_j does not affect the steady state configuration of the model, i.e., it does not affect x_j^* and q_j^*, and therefore it does not affect z_j^* and g_j^*. As we discussed, the reason is that changes in η_j are fully absorbed by the adjustment of the two stocks Z_j and S_j, which in equilibrium comove to produce the constant value q_j^* in equation (31) that does not depend on η_j. In this microfounded extension that yields the *endogenous* variable $p_{D_{ij}}$, the property still applies: changes in fundamentals that change the value of the composite function $C_j(w, p_f) C_{D_j}(R^r, R^w)$ have only transitional effects, like those illustrated in Figure 4; they do not affect the steady-state values x_j^*, q_j^*, z_j^* and g_j^*. However, they affect the industry-specific composition of transportation described by the cost share functions $c_j^r(R^w/R^r)$ and $c_j^w(R^w/R^r)$ and the equilibrium of the fuel market, to which we now turn.

7.4 The price of fuel

To add more richness to the characterization of transportation, we develop a simple model of the market for fuel that nests two equally plausible and interesting scenarios. We assume that the economy has an endowment, F, of a resource that can be processed into fuel. In line with many contributions in this literature (e.g., Peretto and Valente 2011 and 2015, Peretto 2025), we can think of this endowment as constant over time and producing a constant flow of one unit of fuel per unit of time. In other words, the supply of fuel is inelastic and constant at F. An alternative is to allow for a labor cost of extracting and processing the resource into fuel. To keep things as simple as possible, we assume that producing one unit of fuel requires $\varkappa$ units of labor. It follows that the supply of fuels is infinitely elastic at the price $p_f = \varkappa w$ up to the endowment constraint, F, where it becomes completely inelastic.

Figure 9 represents the fuel market that results from this characterization. From the previous subsection, we have that the aggregate demand for fuel is $p_f F = \kappa y^* L$. If this demand is sufficiently low, i.e., it intersects the constraint F at a price below $\varkappa w$, we have scenario 1 in which the price of fuel in general equilibrium (recall the choice of numeraire $w = 1$) is $p_f = \varkappa$, aggregate fuel use is $F^* = \kappa y^* L / \varkappa < F$ and the per unit shipping cost in industry j is

$$p_{D_j} = S_j^{-\sigma_j} C_j \left(1, \varkappa\right) C_{D_j} \left(R^r, R^w\right).$$

If the demand for fuel intersects the constraint F at a price above $\varkappa w$, we have scenario 2 in which the price of fuel in general equilibrium is $p_f = \kappa y^* L / F$, aggregate fuel use is F and the per unit shipping cost is

$$p_{D_j} = S_j^{-\sigma_j} C_j \left(1, \frac{\kappa y^* L}{F}\right) C_{D_j} \left(R^r, R^w\right).$$

In either scenario, the per unit cost of shipping is determined by a self-contained set of intratemporal equilibrium conditions.

This structure highlights the key role of the population, L. If it is constant, we simply have two different equilibria, one with low demand and one with high demand. If population grows at rate λ, we have the third scenario, in which the ratio L/F grows at rate λ and the economy always transitions from the equilibrium in the first scenario to the equilibrium in the second scenario but with constant growth of the price of fuel, i.e., $\dot{p}_f / p_f = \lambda$. In this case, innovation in transportation eventually must contend with the growth of the price of fuel due to population growth. Specifically, we have

$$p_{D_j} = S_j^{-\sigma_j} C_j \left(1, \frac{\kappa y^* L}{F}\right) C_{D_j} \left(R^r, R^w\right) = S_j^{-\sigma_j} L^{1-\omega_j} \vartheta_j \left(\frac{\kappa y^*}{F}\right)^{1-\omega_j} C_{D_j} \left(R^r, R^w\right).$$

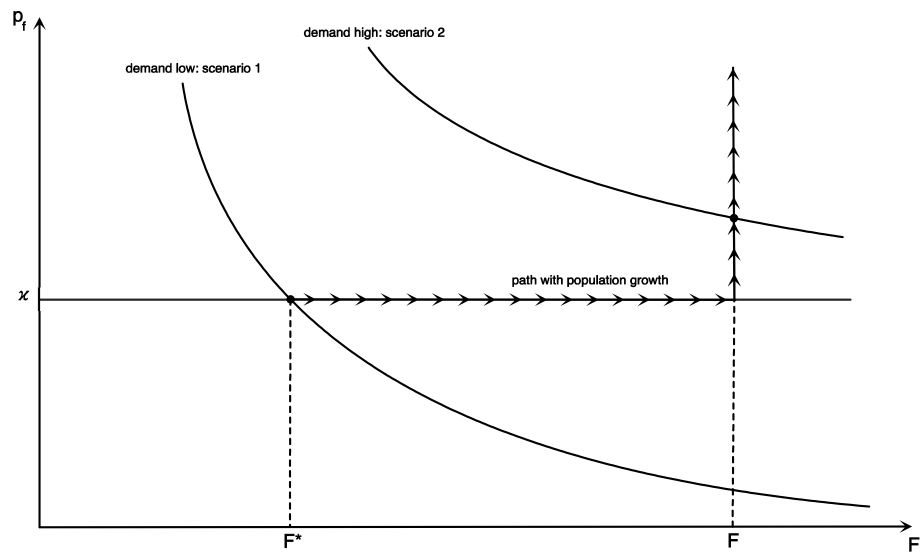

Figure 9: Extended model, equilibrium of the fuel market in three scenarios.

Therefore, we have

$$q_j = \frac{S_j^{-\sigma_j} C_j\left(w, p_f\right) C_{D_j}\left(R^r, R^w\right)}{Z_j^{-\theta} C_j\left(w, p_f\right)} = S_j^{-\sigma_j} Z_j^{\theta_j} \cdot C_{D_j}\left(R^r, R^w\right),$$

which yields

$$\frac{\dot{q}_j}{q_j} = -\sigma_j \frac{\dot{S}_j}{S_j} + \theta_j \frac{\dot{Z}_j}{Z_j}.$$

This equation says that the dynamics do not change. We can then perform comparative dynamics exercises like those in Section 6 by noting that an increase in either R^r or R^w yields an increase in the endogenous variable $C_{D_j}\left(R^r, R^w\right)$, which in the dynamics plays the same role as an increase in the exogenous parameter η_j. What changes in this richer model is that the per unit cost of transportation follows the equation

$$\frac{\dot{p}_{D_j}}{p_{D_j}} = -\sigma_j \frac{\dot{S}_j}{S_j} + \left(1 - \omega_j\right) \frac{\dot{p}_f}{p_f} = -\sigma_j \frac{\dot{S}_j}{S_j} + \left(1 - \omega_j\right) \lambda.$$

In steady state, we thus have

$$\left(\frac{\dot{p}_{D_j}}{p_{D_j}}\right)^* = -\sigma_j s_j^* + \left(1 - \omega_j\right) \lambda.$$

This rate can be positive or negative depending on the magnitude of s_j^*. Moreover, the relation between the knowledge stocks now is $S_j^{-\sigma_j} Z_j^{\theta_j} = q_j^* / C_{D_j}(R^r, R^w)$ and thus depends on the factors, geographical and institutional, that drive R^r and R^w. Industries with lower industry-specific factor $C_{D_j}(R^r, R^w)$ have relatively more distribution-related knowledge, S_j, than production-related knowledge, Z_j.

8 Transportation and secular growth

In this section, we use the richer model of production and transportation that we just developed to study secular growth.

8.1 Phases of growth

Let the typical industry have initial condition $(q_j(0), x_j(0))$, with $x_j(0)$ small and $x_j(0)$ large. The model produces a transition with three phases.

1. Phase I: only variety expansion, $L_{N_j} = 0$, with $L_{Z_j} = L_{S_j} = 0$;

2. Phase II: both variety expansion and cost reduction in transportation, $L_{N_j} > 0$ and $L_{S_j} > 0$, with $L_{Z_j} = 0$;

3. Phase III: all three types of innovation.

Accordingly, along the equilibrium path two phase transitions occur. The first is the onset of systematic innovation in shipping. The second is the onset of systematic innovation in the factory. The valued added of this dynamic structure is that it produces a sequence of events that sheds light on the historical process of development and convergence to modern growth of the typical unit j of the model, which so far we have called an industry but that one can also intepret as a country or as a region within a country. As we have shown, moreover, the model aggregates nicely so that it is straightforward to infer from the dynamics of the individual unit — industry, region — the qualitative dynamics of the aggregate economy. We have already discussed the Phase III dynamics. Here we focus on the dynamics in the first two phases and then bring all the elements together in two comparative dynamics exercises.

8.2 Phase I dynamics

Since $L_{S_j} = L_{Z_j} = 0$, the equations developed in Section 3.3 reduce to the expression for the rate of return to entry in the form

$$\rho = \frac{1}{\beta x_j}\left(\frac{x_j}{\epsilon_j} - \phi_j\right) + \frac{\dot{x}_j}{x_j} - \delta_j.$$

Therefore, the equilibrium of the economy consists of the self-contained process of firm size growth

$$\dot{x}_j = \frac{\phi_j}{\beta} - \left(\frac{1}{\beta\epsilon_j} - \rho - \delta_j\right) x_j,$$

which has analytical solution

$$x_j\left(t\right) = x_j\left(0\right) e^{-\bar{\nu}_j^I t} + \left(1 - e^{-\bar{\nu}_j^I t}\right) \bar{x}_j^I,$$

where:

$$\bar{\nu}_j^I \equiv \frac{1}{\beta\epsilon_j} - \rho - \delta_j \quad and \quad \bar{x}_j^I \equiv \frac{\phi_j}{\frac{1}{\epsilon_j} - \beta\left(\rho + \delta_j\right)}.$$

These are, respectively, the eigenvalue and the steady state of the Phase 1 dynamics.

This solution allows us to compute the time of the first phase transition, the onset of innovation in shipping. To do so, we note that the boundary of the region where $L_{S_j} = 0$ is

$$x_j = \frac{\epsilon_j\left(\rho + \delta_j\right)}{\varsigma_j\sigma_j(\epsilon_j - 1)}\frac{1 + q_j}{q_j}.$$

Since in this phase there is no firm innovation, the variable q_j remains constant at the initial value

$$q_j\left(0\right) = \frac{\left[Z_j\left(0\right)\right]^{\theta_j}}{\left[S_j\left(0\right)\right]^{\sigma_j}} \cdot C_{D_j}\left(R^r, R^w\right)$$

while firm size grows. It follows that the first phase transition occurs when

$$x_j\left(t\right) = \frac{\epsilon_j\left(\rho + \delta_j\right)}{\varsigma_j\sigma_j(\epsilon_j - 1)}\frac{1 + q_j\left(0\right)}{q_j\left(0\right)},$$

which using the solution above yields

$$t^{II} = \frac{1}{\bar{\nu}_j^I}\log\left(\frac{\bar{x}_j^I - x_j\left(0\right)}{\bar{x}_j^I - \frac{\epsilon_j\left(\rho+\delta_j\right)}{\varsigma_j\sigma_j\left(\epsilon_j-1\right)}\frac{1+q_j\left(0\right)}{q_j\left(0\right)}}\right). \tag{37}$$

This handy expression says that t^{II} is decreasing in $q_j(0)$. Therefore, industries or countries with worse productivity of labor and fuel in transportation due to geography and/or institutions start innovating in transportation earlier.

In this phase, the iceberg transportation cost accounts for the fraction

$$\frac{\tau_j}{\tau_j + (1+\tau_j)\, q_j(0)}$$

of the unit expenditure on transportation and transportation accounts for the fraction

$$\frac{\tau_j + (1+\tau_j)\, q_j(0)}{(1+\tau_j)(1+q_j(0))}$$

of the total unit cost of delivering the good to the customer. The employment structure is:

$$L_{X_j} = \frac{\epsilon_j - 1}{\epsilon_j} \frac{x_j}{(1+\tau_j)(1+q_j(0))} + \phi_j;$$

$$L_{D_j} = \frac{\epsilon_j - 1}{\epsilon_j} \frac{\tau_j + (1+\tau_j)\, q_j(0)}{(1+\tau_j)(1+q_j(0))} x_j;$$

$$L_{N_j} = \left(\lambda + \frac{1}{\epsilon_j \beta} - \frac{\phi_j}{\beta x_j} - \rho\right) \beta \varphi_j y^* L.$$

These are all increasing over time as x_j grows due to the fact that in equilibrium population growth is faster than the net entry rate.

8.3 Phase II dynamics

As stated above, the first transition happens at time t_j^{II}. The industry activates innovation in transportation and experiences an acceleration of the growth rate from

$$g_j^I = \chi_j n_j^I = \chi_j \left[\lambda + \frac{1}{\epsilon_j \beta} - \frac{\phi_j}{\beta x_j} - \rho - \delta_j\right]$$

to

$$g_j^{II} = \chi_j n_j^{II} - \frac{q_j}{1+q_j} \frac{\dot{q}_j}{q_j}.$$

In particular, we prove in the appendix that

$$g_j^{II} - g_j^I = \sigma_j \frac{q_j}{1+q_j} \left[\frac{\epsilon_j - 1}{\epsilon_j} \varsigma_j \sigma_j \frac{q_j}{1+q_j} x_j - (\rho + \delta_j)\right] - \frac{\chi_j}{\beta} \left[\frac{q_j}{1+q_j} \frac{\sigma_j(\epsilon_j - 1)}{\epsilon_j} - \frac{\rho + \delta_j}{\varsigma_j x_j}\right] > 0.$$

Since in this phase $L_{Z_j}=0$, the equations developed in Section 5 reduce to the expression for the rate of return to entry in the form

$$\rho=\frac{1}{\beta x_j}\left(\frac{x_j}{\epsilon_j}-\phi_j-L_{S_j}\right)+\frac{\dot{x}_j}{x_j}-\delta_j,$$

where

$$L_{S_j}=\frac{\sigma_j(\epsilon_j-1)}{\epsilon_j}\frac{q_j}{1+q_j}x_j-\frac{\rho+\delta_j}{\varsigma_j}.$$

Therefore, the equilibrium of the industry consists of the the pair of differential equations:

$$\frac{\dot{x}_j}{x_j}=\rho+\delta_j-\frac{1}{\beta\epsilon_j}+\frac{\sigma_j(\epsilon_j-1)}{\beta\epsilon_j}\frac{q_j}{1+q_j}+\frac{1}{\beta x_j}\left(\phi_j-\frac{\rho+\delta_j}{\varsigma_j}\right); \tag{38}$$

$$\frac{\dot{q}_j}{q_j}=\sigma_j\left(\rho+\delta_j-\varsigma_j\sigma_j\frac{\epsilon_j-1}{\epsilon_j}\frac{q_j}{1+q_j}x_j\right). \tag{39}$$

This 2D system has no analytical solution. Thus, the best that we can do is characterize the dynamics qualitatively.

Since in this phase q_j falls throughout, the iceberg transportation cost accounts for the *rising* fraction

$$\frac{\tau_j}{\tau_j+(1+\tau_j)\,q_j\,(t)}$$

of the unit expenditure on transportation and transportation accounts for the *falling* fraction

$$\frac{\tau_j+(1+\tau_j)\,q_j}{(1+\tau_j)\,(1+q_j)}$$

of the total unit cost of delivering the good to the customer. The employment structure is:

$$L_{X_j}=\frac{\epsilon_j-1}{\epsilon_j}\frac{x_j}{(1+\tau_j)\,(1+q_j)}+\phi_j;$$

$$L_{D_j}=\frac{\epsilon_j-1}{\epsilon_j}\frac{\tau_j+(1+\tau_j)\,q_j}{(1+\tau_j)\,(1+q_j)}x_j;$$

$$L_{S_j}=\frac{\sigma_j(\epsilon_j-1)}{\epsilon_j}\frac{q_j}{1+q_j}x_j-\frac{\rho+\delta_j}{\varsigma_j},$$

$$L_{N_j}=\left[\lambda+\frac{1}{\beta\epsilon_j}-\frac{\sigma_j(\epsilon_j-1)}{\beta\epsilon_j}\frac{q_j}{1+q_j}-\frac{1}{\beta x_j}\left(\phi_j-\frac{\rho+\delta_j}{\varsigma_j}\right)-\rho\right]\beta\varphi_j y^* L.$$

Differently from Phase I, the movement over time of q_j either reinforces of counteracts the positive effect of the growing firm size on each form of employment. In particular, L_{X_j} and L_{N_j} grow faster because the falling q_j reinforces the effect of the rising sales of the firm; L_{D_j} and L_{S_j}, instead, grow slower because the falling q_j counteracts the effect of the rising sales of the firm.

8.4 Two comparative exercises

To extract more insight from the extended model, in this section we do two comparative dynamics exercises that shed light on different properties of the framework that this paper proposes.

Comparative secular development dynamics. Figure 10 illustrates the dynamics of two industries or countries that differ only by the initial value $q_j(0)$ due to differences in geography, regulations or the other factors microfounded in Section 7. For simplicity and concreteness, henceforth we refer to countries. At time $t = 0$, the red country has higher transportation cost than the black country. Both countries move up vertically as their respective populations grow and drive firm size growth. The red country crosses the boundary $L_{S_j} = 0$ earlier and at smaller firm size than the black country; see the analytical solution (37) above. The reason is that the country starts closer to the decreasing activation boundary because of its higher initial value $q_j(0)$. The intuition, therefore, is that the threshold of firm size that activates innovation in transportation is lower the lower the productivity of labor and fuel in shipping things relative to the productivity of labor and fuel in making things. Equivalently, we can say that for given firm size (volume of sales), the return to lowering the cost of shipping things is higher the higher the cost of shipping things. Once each country crosses the boundary $L_{S_j} = 0$ and starts reducing the shipping cost, the dynamics are governed by the two nonlinear differential equations discussed above. The phase diagram says that, intuitively, the red country activates innovation in production later and at larger firm size than the black country. The intuition is similar to the activation of innovation in transportation. The boundary $L_{Z_j} = 0$ is increasing in q_j because the return to lowering the cost of production is higher the higher the productivity of labor and fuel in shipping things relative to the productivity of labor and fuel in making things. One way to see this, is that the lower the cost of shipping goods, the higher the marginal profit from increasing the volume of production via a marginal reduction of the cost of making those goods. By construction, eventually the two countries converge to the same steady state $\left(q_j^*, x_j^*\right)$.

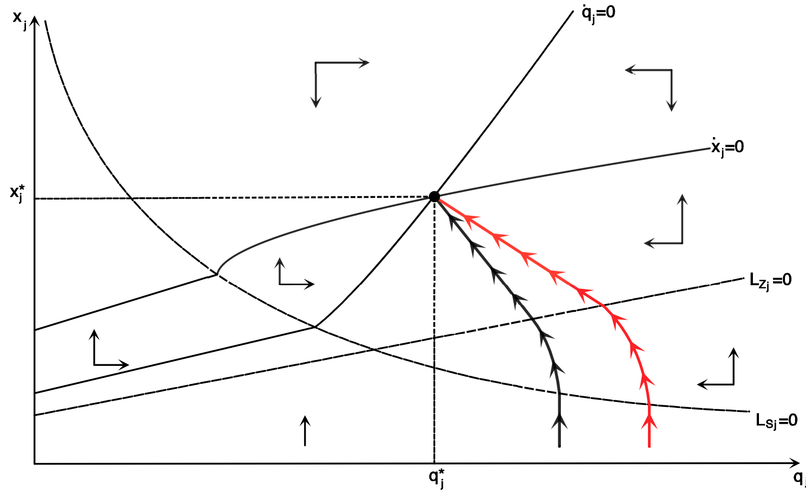

Figure 10: Comparative secular development dynamics: two industries or countries that differ only by the initial value $q_j(0)$ due to differences in geography and institutions.

Counterfactual: no transportation or no innovation in transportation. Figure 11 illustrates the dynamics of one industry or country under the dynamics of the model with innovation in transportation, in black, and the dynamics under the counterfactual model, in red, that either has no transportation or that has no innovation in transportation. This exercise compares theories: the first counterfactual represents the dynamics of a Schumpeterian model of innovation-led endogenous growth that ignores the distinction and complementarity between making things and shipping things. This is pretty much all of growth economics to date. The second counterfactual represents the dynamics of a model that acknowledges the importance of transportation but fails to allow for innovation in transportation. The key point of the comparison is that the dynamics suggest that innovation in transportation is a catalyst for innovation in manufacturing in the sense that it hastens the second phase transition. Specifically, the black equilibrium trajectory crosses the boundary $L_{Z_j} = 0$ and activates innovation in making things earlier and at smaller firm size than the red trajectory. By construction, both trajectories converge to the steady state $\left(q_j^*, x_j^*\right)$.

9 Conclusion

In this paper, we have developed a tractable multi-industry model of endogenous growth in which per-unit transportation costs play a central role in shaping innovation incentives and aggregate dynamics. By explicitly modeling innovation in both manufacturing and

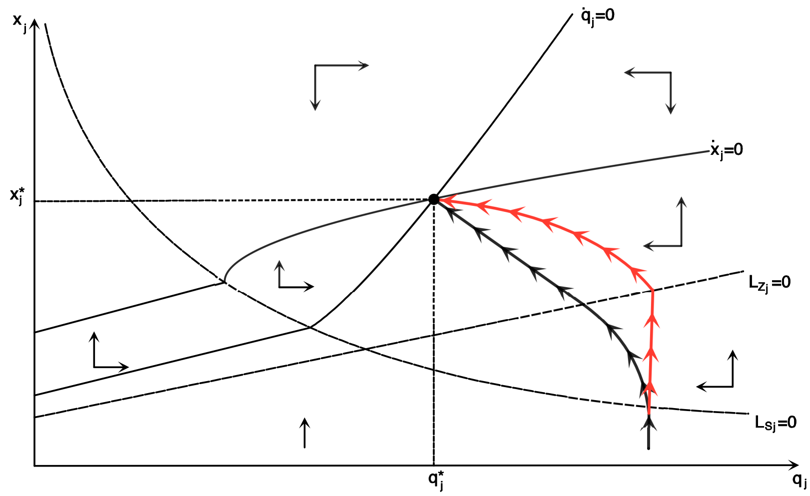

Figure 11: Counterfactual: one industry or country in the model with innovation in transportation vs. the model with no innovation in transportation or no transportation altogether.

transportation, we have shown that innovation in transportation is a necessary complement to process innovation in production. When the cost of shipping goods is exogenous, the sales amplification of manufacturing productivity improvements eventually peters out and growth ceases. When firms allocate R&D resources to endogenous innovation in transportation, a balanced-growth equilibrium emerges in which cost-reducing innovation in both making things and shipping things sustains long-run expansion.

The analysis shows that the economy's growth potential depends not only on firms' ability to improve production efficiency but also on technological advances in distribution. Policies or frictions that raise per-unit transportation costs — such as cumulative regulation, infrastructure bottlenecks, or fuel price pressures — dampen the translation of productivity gains in the factory into aggregate growth. Conversely, innovations that lower per-unit transportation costs or improve logistical efficiency generate both level and growth effects by stranghtening the ability of firms to turn process innovations into a larger volume of sales.

More broadly, the results suggest that understanding growth requires treating production and distribution as an integrated system. Future research could extend the framework to include quality improvements, endogenous infrastructure investment, or international linkages, thereby providing a richer theory of how innovation and transportation jointly determine long-run prosperity.

10 Appendix

10.1 The dynamic decisions of the firm

Because the Hamiltonian is linear in $L_{Z_{ij}}$ and in $L_{S_{ij}}$, the investment decisions have a bang-bang structure. For $w < v_{Z_{ij}}\alpha_j K_j$, the value of the marginal unit of manufacturing knowledge is higher than its cost. Then, every agent in the economy wants to quit alternative employment to work in firm i's manufacturing-specific R&D. This allocation of labor cannot be an equilibrium. For $w > v_{Z_{ij}}\alpha_j K_j$, the value of the marginal unit of manufacturing knowledge is lower than its cost of production. The firm then does not invest in manufacturing knowledge accumulation and we have the corner solution $L_{Z_{ij}} = 0$. For $w = v_{Z_{ij}}\alpha_j K_j$, we have the interior solution given by the equality between marginal revenue and marginal cost of manufacturing knowledge accumulation. Similarly, for $w < v_{S_j}\varsigma_j S_j$, the value of the marginal unit of transportation knowledge is higher than its cost. Then, every agent in the economy wants to quit alternative employment to work in firm i's transportation-specific R&D. This allocation of labor cannot be an equilibrium. For $w > v_{S_{ij}}\varsigma_j S_j$, the value of the marginal unit of transportation knowledge is lower than its cost of production. The firm then does not invest in transportation knowledge accumulation and we have the corner solution $L_{S_{ij}} = 0$. For $w = v_{S_{ij}}\varsigma_j S_j$, we have the interior solution given by the equality between marginal revenue and marginal cost of transportation knowledge accumulation.

Finally, value maximization yields the following pair of differential equation for the costate variables:

$$r + \delta_j = \theta_j w (1 + \tau_j) Z_i^{-\theta_j - 1} \frac{X_{ij}}{v_{Z_{ij}}} + \frac{\dot{v}_{Z_{ij}}}{v_{Z_{ij}}}; \tag{40}$$

$$r + \delta_j = \sigma_j w (1 + \tau_j) \eta_j S_{ij}^{-\sigma_j - 1} \frac{X_{ij}}{v_{S_{ij}}} + \frac{\dot{v}_{S_{ij}}}{v_{S_{ij}}}. \tag{41}$$

Associated to these equations are the terminal conditions:

$$\lim_{s \to \infty} e^{-\int_t^s [r(v) + \delta_j] dv} v_{S_{ij}}(s) S_{ij}(s) = 0;$$

$$\lim_{s \to \infty} e^{-\int_t^s [r(v) + \delta_j] dv} v_{Z_{ij}}(s) Z_{ij}(s) = 0.$$

Equation (40) defines the rate of return to manufacturing knowledge accumulation as the ratio between revenues from the manufacturing knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of manufacturing knowledge. The revenue from the marginal unit of manufacturing knowledge is the cost reduction it yields times the scale of production to which it applies. Similarly, equation (41) defines the rate of return transportation knowledge accumulation as the ratio between revenues from the transportation knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of transportation knowledge. The revenue from the marginal unit of transportation knowledge is the cost reduction it yields times the scale of production to which it applies. Using the price strategy (14) and solving forward these differential equations, we obtain the value of each unit of output of the two R&D units as the present discounted value of the revenue from the marginal cost reduction:

$$v_{Z_{ij}}(t) = \int_t^{\infty} e^{-\int_t^s [r(v)+\delta_j]dv} \frac{\theta_j(\epsilon_j - 1)}{\epsilon_j} \frac{p_{X_{ij}}(s) X_{ij}(s)}{Z_{ij}(s)} ds;$$

$$v_{S_{ij}}(t) = \int_t^{\infty} e^{-\int_t^s [r(v)+\delta_j]dv} \frac{\sigma_j(\epsilon_j - 1)}{\epsilon_j} \frac{p_{X_{ij}}(s) X_{ij}(s)}{Z_{ij}(s)} ds.$$

Accordingly, the total value produced by the manufacturing innovation unit is $v_{Z_{ij}} Z_{ij}$ and the total value produced by the transportation innovation unit is $v_{S_{ij}} S_{ij}$.

We now take logs and time-derivative of the indifference conditions $w = v_{Z_{ij}} \alpha_j Z_j$ and $w = v_{S_{ij}} \varsigma_j S_j$, and use the demand curve (7), the price equation (14), the firm innovation technologies (10) and (11), and symmetry, to reduce equations (40) and (41) to:

$$r = r_{Z_j} \equiv \alpha_j \left[\frac{Y_j}{wN_j} \frac{\theta_j(\epsilon_j - 1)}{\epsilon_j} \frac{Z_{ij}^{-\theta_j}}{Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}} - L_{Z_j} \right] + \frac{\dot{w}}{w} - \delta_j;$$

$$r = r_{S_j} \equiv \varsigma_j \left[\frac{Y_j}{wN_j} \frac{\sigma_j(\epsilon_j - 1)}{\epsilon_j} \frac{\eta_j S_{ij}^{-\sigma_j}}{Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}} - L_{S_j} \right] + \frac{\dot{w}}{w} - \delta_j.$$

These are the expressions reported in the main text.

10.2 Construction of the phase diagram in section 5

We have two cases: $\theta_j > \sigma_j$ and $\theta_j < \sigma_j$. We do only the first since the second yields similar results. The steady-state system is:

$$\dot{q}_j \geq 0 : \quad x_j \geq x_j(q_j)_{\dot{q}_j = 0} \equiv \frac{(\theta_j - \sigma_j)(\rho + \delta_j)(1 + q_j)}{\frac{\epsilon_j - 1}{\epsilon_j}\left(\alpha_j \theta_j^2 - \varsigma_j \sigma_j^2 q_j\right)};$$

$$\dot{x}_j \geq 0: \quad x_j \leq x_j(q_j)_{\dot{x}_j=0} \equiv \frac{\left[\phi_j - (\rho+\delta_j)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)\right](1+q_j)}{\frac{1}{\epsilon_j} - \beta(\rho+\delta_j) + \frac{\epsilon_j-1}{\epsilon_j}\theta_j + \left[\frac{1}{\epsilon_j} - \beta(\rho+\delta_j) - \frac{\epsilon_j-1}{\epsilon_j}\sigma_j\right] q_j}.$$

The $\dot{q}_j = 0$ locus starts at

$$x_j(0)_{\dot{q}_j=0} = \frac{(\theta_j - \sigma_j)(\rho+\delta_j)\epsilon_j}{(\epsilon_j - 1)\alpha_j\theta_j^2}$$

and is increasing in q_j with a vertical asymptote at

$$q_j = \frac{\alpha_j\theta_j^2}{\varsigma_j\sigma_j^2}.$$

The $\dot{x}_j = 0$ locus starts at

$$x_j(q_j)_{\dot{x}_j=0} = \frac{\phi_j - (\rho+\delta_j)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)}{\frac{1}{\epsilon_j} - \beta(\rho+\delta_j) + \frac{\epsilon_j-1}{\epsilon_j}\theta_j}$$

and is increasing in q_j with a horizontal asymptote at

$$\lim_{q_j\to\infty} x_j(q_j)_{\dot{x}_j=0} = \frac{\phi_j - (\rho+\delta_j)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)}{\frac{1}{\epsilon_j} - \beta(\rho+\delta_j) + \frac{\epsilon_j-1}{\epsilon_j}\sigma_j}.$$

Therefore, there is a unique steady state for

$$\frac{\phi_j - (\rho+\delta_j)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)}{\frac{1}{\epsilon_j} - \beta(\rho+\delta_j) + \frac{\epsilon_j-1}{\epsilon_j}\theta_j} > \frac{\epsilon_j(\theta_j - \sigma_j)(\rho+\delta_j)}{\alpha_j(\epsilon_j - 1)\theta_j^2}$$

and there are two steady states for

$$\frac{\phi_j - (\rho+\delta_j)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)}{\frac{1}{\epsilon_j} - \beta(\rho+\delta_j) + \frac{\epsilon_j-1}{\epsilon_j}\theta_j} \leq \frac{\epsilon_j(\theta_j - \sigma_j)(\rho+\delta_j)}{\alpha_j(\epsilon_j - 1)\theta_j^2}.$$

However, the smaller steady state is unstable (a saddle point). It follows that we can study the model with only one steady state without loss of generality.

10.3 Derivation of the welfare equation

We write the log of TFP in the form

$$\begin{aligned} \log T_j &= \theta_j \int_0^t z_j(k)\,dk + \chi_j \log \frac{N_j}{Y_j} + \chi_j \log Y_j - \log(1+q_j) \\ &= \theta_j \int_0^t z_j(k)\,dk - \chi_j \log x_j - \log(1+q_j) + \varkappa + \chi_j \lambda t, \end{aligned}$$

where

$$\varkappa \equiv \chi_j \log \varphi_j + \chi_j \log y^* + \chi_j \log \Lambda.$$

We approximate the log terms to write

$$\log T_j \simeq \theta_j \int_0^t z_j(k)\,dk - \chi_j x_j - q_j + \varkappa + \chi_j \lambda t.$$

Next, we linearize around the new steady state. In particular, we write:

$$z_j = \gamma_0 + \gamma_x x_j - \gamma_q q_j;$$

$$x_j(t) = x_j(0)\,e^{-\nu_{x_j} t} + x_j^* \left(1 - e^{-\nu_{x_j} t}\right) = \left[x_j(0) - x_j^*\right] e^{-\nu_{x_j} t} + x_j^* = \Delta_{x_j} e^{-\nu_{x_j} t} + x_j^*;$$

$$q_j(t) = q_j(0)\,e^{-\nu_{x_j} t} + q_j^* \left(1 - e^{-\nu_{x_j} t}\right) = \left[q_j(0) - q_j^*\right] e^{-\nu_{q_j} t} + q_j^* = \Delta_{q_j} e^{-\nu_{q_j} t} + q_j^*.$$

Now we work on the various terms separately. We start with

$$\begin{aligned} \int_0^t z_j(k)\,dk &= \int_0^t \left[\gamma_{j0} + \gamma_{jx} x_j(k) - \gamma_{jq} q_j(k)\right] dk \\ &= \int_0^t \left(\gamma_{j0} + \gamma_{jx} x_j^* - \gamma_{jq} q_j^*\right) dk + \int_0^t \gamma_{jx} \Delta_{x_j} e^{-\nu_{x_j} k} dk - \int_0^t \gamma_{jq} \Delta_{q_j} e^{-\nu_{q_j} k} dk \\ &= z_j^* t + \gamma_{jx} \Delta_{x_j} \int_0^t e^{-\nu_x k} dk - \gamma_{jq} \Delta_{q_j} \int_0^t e^{-\nu_{q_j} k} dk \\ &= z_j^* t + \gamma_{jx} \Delta_{x_j} \left(1 - e^{-\nu_{x_j} t}\right) - \gamma_{jq} \Delta_{q_j} \left(1 - e^{-\nu_{q_j} t}\right). \end{aligned}$$

We then write in compact notation

$$\begin{aligned} \log T_j(t) &= \theta_j \Delta_{x_j} - \theta_j \Delta_{q_j} - \left(\theta_j + \chi_j\right) \Delta_{x_j} e^{-\nu_x t} + \left(\theta_j + \chi_j\right) \Delta_{q_j} e^{-\nu_q t} + \varkappa + \chi_j \lambda t \\ &= \theta_j \left(\Delta_{x_j} - \Delta_{q_j}\right) - \left(\theta_j + \chi_j\right) \left(\Delta_{x_j} e^{-\nu_x t} - \Delta_{q_j} e^{-\nu_q t}\right) + \varkappa + \chi_j \lambda t. \end{aligned}$$

Finally, we have

$$\frac{U^*}{\Lambda} = \int_0^\infty e^{-(\rho-\lambda)t} \log\left(\frac{y^*}{c^*}\right) dt + \Gamma^*,$$

where

$$\begin{aligned}
\Gamma^* &= \sum_{j=1}^J \varphi_j \left[\int_0^\infty e^{-(\rho-\lambda)t} \left[\varkappa + g_j^* t + \theta_j \left(\Delta_{x_j} - \Delta_{q_j}\right) - \left(\theta_j + \chi_j\right)\left(\Delta_{x_j} e^{-\nu_x t} - \Delta_{q_j} e^{-\nu_q t}\right)\right] dt\right] \\
&= \sum_{j=1}^J \varphi_j \left[\int_0^\infty e^{-(\rho-\lambda)t} \varkappa dt\right] + \sum_{j=1}^J \varphi_j \left[\int_0^\infty e^{-(\rho-\lambda)t} g_j^* t dt\right] \\
&\quad + \sum_{j=1}^J \varphi_j \left[\int_0^\infty e^{-(\rho-\lambda)t} \theta_j \left(\Delta_{x_j} - \Delta_{q_j}\right) dt\right] \\
&\quad - \sum_{j=1}^J \varphi_j \left[\int_0^\infty e^{-(\rho-\lambda)t} \left(\theta_j + \chi_j\right) \Delta_{x_j} e^{-\nu_x t} dt\right] \\
&\quad + \sum_{j=1}^J \varphi_j \left[\int_0^\infty e^{-(\rho-\lambda)t} \left(\theta_j + \chi_j\right) \Delta_{q_j} e^{-\nu_q t} dt\right].
\end{aligned}$$

Collecting and rearranging terms, we reduce this expression to

$$\Gamma^* = \frac{\varkappa}{\rho-\lambda} + \sum_{j=1}^J \frac{\varphi_j g_j^*}{(\rho-\lambda)^2} + \sum_{j=1}^J \frac{\varphi_j}{\rho-\lambda} \underbrace{\left[\theta_j \left(\Delta_{x_j} - \Delta_{q_j}\right) - \frac{\left(\theta_j + \chi_j\right)\Delta_{x_j}}{\rho - \lambda + \nu_{x_j}} + \frac{\left(\theta_j + \chi_j\right)\Delta_{q_j}}{\rho - \lambda + \nu_{q_j}}\right]}_{\Upsilon_j}.$$

The last term in this expression is the cumulative change in TFP due to the transition from the initial point $\left(q_j^o, x_j^o\right)$ to the new steady state $\left(q_j^*, x_j^*\right)$.

Therefore, we can integrate and write welfare per capita

$$\frac{U^*}{\Lambda} = \frac{\log\left(\frac{y^*}{c^*}\right) + \varkappa}{\rho - \lambda} + \sum_{j=1}^J \frac{\varphi_j g_j^*}{(\rho-\lambda)^2} + \sum_{j=1}^J \frac{\varphi_j \Upsilon_j}{\rho - \lambda}.$$

10.4 Proof that growth accelerates in Phase II

At the onset of Phase II, the growth rate changes from

$$g_j^I = \chi_j n_j^I = \chi_j \left[\lambda + \frac{1}{\epsilon_j \beta} - \frac{\phi_j}{\beta x_j} - \rho - \delta_j\right]$$

to

$$
\begin{aligned}
g_j^{II} &= \chi_j n_j^{II} - \frac{q_j}{1+q_j}\frac{\dot{q}_j}{q_j} \\
&= \chi_j \left[\lambda + \frac{1}{\epsilon_j \beta} - \frac{\phi_j - \frac{\rho+\delta_j}{\varsigma_j}}{\beta x_j} - \frac{\sigma_j q_j}{1+q_j}\frac{\epsilon_j - 1}{\epsilon_j \beta} - \rho - \delta_j\right] \\
&\quad + \frac{q_j}{1+q_j}\left[\frac{\epsilon_j - 1}{\epsilon_j}\varsigma_j \sigma_j^2 \frac{q_j}{1+q_j} x_j - \sigma_j\left(\rho+\delta_j\right)\right]. \\
&= \chi_j \left[\lambda + \frac{1}{\epsilon_j \beta} - \frac{\phi_j}{\beta x_j} - \rho - \delta_j\right] \\
&\quad + \frac{\chi_j}{\beta}\left[\frac{\frac{\rho+\delta_j}{\varsigma_j}}{x_j} - \frac{q_j}{1+q_j}\frac{\sigma_j\left(\epsilon_j - 1\right)}{\epsilon_j}\right] \\
&\quad + \frac{q_j}{1+q_j}\left[\frac{\epsilon_j - 1}{\epsilon_j}\varsigma_j \sigma_j^2 \frac{q_j}{1+q_j} x_j - \sigma_j\left(\rho+\delta_j\right)\right]
\end{aligned}
$$

To show that this is positive, recall that

$$
s_j = \varsigma_j L_{S_j} = \frac{\varsigma_j \sigma_j (\epsilon_j - 1)}{\epsilon_j}\frac{q_j}{1+q_j} x_j - \rho - \delta_j,
$$

where $L_{S_j} > 0$ for

$$
x_j > x_{S_j}\left(q_j\right) \equiv \frac{\epsilon_j\left(\rho+\delta_j\right)}{\varsigma_j \sigma_j (\epsilon_j - 1)}\frac{1+q_j}{q_j}.
$$

We write the expression above

$$
g_j^{II} = \chi_j n_j^{I} + \underbrace{\frac{\chi_j}{\beta}\left[\frac{\rho+\delta_j}{\varsigma_j x_j} - \frac{q_j}{1+q_j}\frac{\sigma_j\left(\epsilon_j - 1\right)}{\epsilon_j}\right]}_{\text{negative when } L_{S_j}>0} + \underbrace{\frac{q_j}{1+q_j}\left[\frac{\epsilon_j - 1}{\epsilon_j}\varsigma_j \sigma_j^2 \frac{q_j}{1+q_j} x_j - \sigma_j\left(\rho+\delta_j\right)\right]}_{\text{positive when } L_{S_j}>0}.
$$

Therefore, we have

$$
g_j^{II} - g_j^{I} = \sigma_j \frac{q_j}{1+q_j}\left[\frac{\epsilon_j - 1}{\epsilon_j}\varsigma_j \sigma_j \frac{q_j}{1+q_j} x_j - \left(\rho+\delta_j\right)\right] - \frac{\chi_j}{\beta}\left[\frac{q_j}{1+q_j}\frac{\sigma_j\left(\epsilon_j - 1\right)}{\epsilon_j} - \frac{\rho+\delta_j}{\varsigma_j x_j}\right] > 0
$$

because holding constant q_j the two components of the right hand side intersect exactly at

$$
x_j = x_{S_j}\left(q_j\right) \equiv \frac{\epsilon_j\left(\rho+\delta_j\right)}{\varsigma_j \sigma_j (\epsilon_j - 1)}\frac{1+q_j}{q_j},
$$

with the property that for $x_j < x_{S_j}(q_j)$ the first term is negative while the second is positive, while for $x_j > x_{S_j}(q_j)$ the first term is positive while the second is negative. In other words, the difference $g_j^{II} - g_j^I$ is zero exactly at $x_j = x_{S_j}(q_j)$ and is negative for $x_j < x_{S_j}(q_j)$ while it is positive for $x_j > x_{S_j}(q_j)$.

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